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Learning by Doing in the Presence of an Open Access Renewable Resource: Is Growth Sustainable?

Carol McAudland
University of Michigan

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Learning By Doing in the Presence of an Open Access Renewable Resource: Is Growth Sustainable?

Carol McAusland
Department of Economics
University of Michigan
#238 - 611 Tappan Hall
Ann Arbor, MI
USA 48109

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Abstract

This paper focuses on the impact of technological progress (modeled as learning by doing) on economic growth when one of the inputs in production is an open access renewable resource. Technological progress is found to indirectly induce resource depletion, such that sustainable growth will not occur in autarky under certain preferences, and is possible in trade only if the resource sector contracts over time or shuts down completely. Comparisons of steady state welfare in autarky and free trade reveal that for very high or low world prices of the resource-based good, it is possible for the economy to gain from trade. However if the price is intermediate, it will instead lose.

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1 Introduction

One of the most compelling characteristics of economic development in the last thirty years has been the rapid economic growth of many "resource poor" countries: Hong Kong, Singapore, Taiwan, South Korea, Japan. At the same time, many regions once rich with forests, fishing grounds, or mineral deposits have experienced slow—sometimes even negative—economic growth: Côte d'Ivoire, Madagascar, Haiti, Nigeria, the Philippines.[1] The most popular explanation for the failure of such countries to transform their resource endowments into material wealth has been the inappropriate use of tariffs and subsidies that accompany an import substitution regime. But even under free trade, would these resource rich regions have prospered?

In this paper I look at the mechanics of economic growth, driven by learning by doing, when one of the goods produced in an economy is the harvest of an open access renewable resource. It is found that under certain preferences economic growth is unsustainable in autarky. In contrast, when open to trade as a Small Open Economy the country may avoid the zero or negative growth trap, but only if its harvesting sector declines over time, or shuts down entirely. If the economy instead specializes in production of the harvested good in trade, then positive economic growth is unsustainable, and the economy may end up with lower steady state utility than in

[1] There are of course countries that defy this pattern and have achieved staples led growth: consider Canada and Australia. However, in at least these two cases, growth was achieved through a series of staples. Lewis (1989) writes "A long sweep of Canadian history saw the prominence of a succession of natural-resource-based exports, ranging from fish and furs to timber, foodstuffs (principally wheat), and minerals..." (p.1575). In Australia, the list of staple exports over the last century includes "whale products, gold, wool, dairy products, meat, coal, base metals, and most recently, diamonds." (Lewis, p.1577)

autarky.

The problem is essentially that technological progress, which raises the rate at which a given workforce can harvest the resource, may thereby lower the sustainable size of the resource stock. When productivity depends on the size of the resource stock (as I assume it does) this resource decline has a negative and offsetting impact on labor productivity, possibly to the extent where overall labor productivity in this sector falls as an indirect result of technological progress. In fact, if technological progress is unlimited, then under certain preferences both resource extinction and continuously declining welfare are inevitable in autarky. If instead there is a dampening force on technological growth (depreciation of human capital, for example) then the autarkic economy will settle into a steady state in both the resource stock and welfare.

Free trade offers two ways to improve welfare. If world relative prices motivate labor to exit the sector dependent on the input of a depletable resource, then positive economic growth may be sustainable. But even if trade induces specialization in resource harvesting, trade may still raise steady state welfare if resource prices are high enough to compensate for losses in overall productivity. However, and as in autarky, when specialized in the harvested good, the highest rate of welfare growth that the trading economy can sustain is zero.

In structure the model in this paper follows that in Brander and Taylor (1997a): the economy is Ricardian with one mobile factor, labor, and there is an open access renewable resource specific to a harvesting sector. This model provides a convenient framework in which to analyze the effects of technological change on an economy's production structure, pattern of resource use, and welfare over time. Other authors who have also considered the interaction between trade and resource use include Brander and Taylor (1997b, Forthcoming(a)), Chichilnisky (1994a, 1994b), Karp, Sacheti and Zhao (1996), Krugman (1987), Matsuyama (1992), Rauscher (1994), and Sachs and Warner (1995). As with the present paper, most of these works contribute to debate on the following questions: does resource abundance slows human capital accumulation, and does trade necessarily lower welfare in countries with open access resources?

The first debate has grown out of the Dutch Disease literature, which focuses on the terms of trade effects of resource booms and asks whether or not resource abundance can make a country worse off. The theoretical models in Matsuyama (1992) and Sachs and Warner (1995) address this question in endogenous growth frameworks, and each makes the critical assumption that only employment in industrial sectors can contribute to human capital accumulation. Consequently, they find that whenever trade or a resource boom raises demand for resource-based or non-tradable goods, such that the number of workers employed in the industrial sector falls, the growth rate of an economy will also fall. However this assumption is problematic as it dismisses the potential for innovation in other resource-based industries, which, perhaps contrary to common belief, can be highly technical at both the harvesting and processing levels of production.[2] In this paper a different approach is taken. The assumption that resource-based production is inherently unable to contribute to human capital accumulation is omitted; instead, attention is focused on the conse-

[2]See, for example, discussions in Dykstra (1997), Scott and Pearse (1992), and Squires (1992).

quences for growth when labor productivity depends not only on the stock of human capital, but also on the stock of a (potentially) depletable natural resource.

The setting for this analysis is an institutional environment common to many developing, and some industrialized, economies: the stock of human capital and the natural resource are both open access. In this context, the welfare effects of trade are analyzed, permitting this paper to also contribute to debate on the second question identified above: does trade lower welfare in countries with open or restricted access resources? As mentioned previously, in this paper two routes are found through which trade may improve welfare: by offering relative prices which compensate for lost productivity, or by inducing labor to exit harvesting. This last route is similarly identified by Brander and Taylor (1997b), who find that trade improves welfare in an economy with open access resources if the resource stock is sufficiently depleted at the moment of trade liberalization. In contrast, in models where the externality associated with open access is modeled as contemporaneous[3] in nature, trade is found to either hurt countries with restricted access resources (Chichilnisky 1994a, 1994b) or have at best non-negative effects (Karp, Sacheti and Zhao, 1996).

The rest of the paper is laid out as follows. In section 2 the basic framework of the model is discussed: preferences, production functions, and the dynamics governing the renewable resource and the stock of human capital. In sections 3 and 4, respectively, the potential for long run growth in autarky and the Small Open Economy, plus

[3] Loosely, models of open access resources incorporate one of two types of externalities: contemporaneous, in which agents' affect one another's current costs of production, and intertemporal, in which total harvest in one period affects the size of the stock available next period.

welfare comparisons, are analyzed. In section 5 the consequences of production and employment taxes are briefly considered, while caveats are discussed in section 5. Section 6 concludes.

2 Model

The basic framework is a continuous time Ricardian economy capable of producing two goods: a numeraire manufactured good M and a harvested good H. The economy is endowed with $\bar{L}$ workers, measured in units such that $\bar{L} = 1$. All consumers are workers (and vice versa) and are assumed to have identical and homothetic preferences over the two goods, such that reference may be made to a representative consumer. In particular, the time t instantaneous utility of the representative consumer is assumed to be Cobb-Douglas:

$$U(H(t), M(t)) = H(t)^a M(t)^{(1-a)}, \qquad a \in (0,1) \tag{1}$$

where $H(t)$ and $M(t)$ are the quantities of H and M consumed at time t. These preferences give rise to momentary Marshallian demands $H^c(t)$ and $M^c(t)$

$$H^c(t) = \frac{aI(t)}{p(t)} \tag{2}$$

$$M^c(t) = (1 \Leftrightarrow a)I(t) \tag{3}$$

and indirect instantaneous utility

$$V(p(t), I(t)) = \frac{\eta I(t)}{p(t)^a} \tag{4}$$

where $\eta = a^a(1 \Leftrightarrow a)^{(1-a)}$ and $I(t)$ is the representative consumer's income at time t.

2.1 Production

As mentioned above the economy is endowed with a perfectly inelastic supply of workers; these workers are perfectly mobile across sectors. The economy also employs an open access renewable resource S that is specific to production of the harvested good. It is assumed that $S \in [0, K]$. Note that, because the resource is open access, all users of the resource do so without payment of any fees. The production functions for each of the two goods at a point in time t are given by

$$H(t) = L_H(t)S(t)f(E(t))$$

$$M(t) = L_M(t)f(E(t))$$

where $L_i(t)$ is the amount of labor employed by sector i at t . The f function represents the efficiency of labor in sector i at time t , which is assumed to be an increasing function of the economy's human capital E. In particular, it is assumed that

$$f(E(t)) = \beta E(t)$$

where

$$E(t) = E_{t_0} + \int_{t_0}^{t} e^{\delta(u-t)}[M(u) + H(u)]du \tag{5}$$

such that the parameter δ is the rate at which human capital depreciates.[4]

Under perfect competition in the markets for L, M and H, it is clear that in this

[4] In equation (5) it is implicitly assumed that M and H are measured in comparable units such that addition of $M(t)$ and $H(t)$ is meaningful.

Ricardian economy,

$$L_H(t) \quad \begin{cases} = & 1 & \text{if} & p(t)S(t) > 1 \\ \in & [0,1] & \text{if} & p(t)S(t) = 1 \\ = & 0 & \text{if} & p(t)S(t) < 1 \end{cases} \tag{6}$$

and $L_M(t) = 1 \Leftrightarrow L_H(t)$. Thus in equilibrium the instantaneous production functions can be rewritten as

$$H(t) = L_H(t)S(t)\beta E(t) \tag{7}$$

$$M(t) = [1 \Leftrightarrow L_H(t)]\beta E(t)\,, \tag{8}$$

and note for later reference that

$$I(t) = p(t)L_H(t)S(t)\beta E(t) + [1 \Leftrightarrow L_H(t)]\beta E(t) \tag{9}$$

$$= \beta E(t)[1 \Leftrightarrow L_H(t)[1 \Leftrightarrow p(t)S(t)]]\,. \tag{10}$$

Moving next to the dynamics of the system, the growth rate of the renewable resource stock S is the difference between the rate of natural growth $G(S)$ and the rate of harvest H. The rate of natural growth is assumed to have the standard logistic form

$$G(S) = gS(t)\left[1 \Leftrightarrow \frac{S(t)}{K}\right]$$

in which g is the intrinsic growth rate of the resource and K is its carrying capacity (which is assumed to be greater than 1). That is, g is the rate of natural growth of the resource as the crowding term $\frac{S}{K}$ approaches zero; K is the level to which the resource stock would converge in the complete absence of harvesting activity. Thus the rate of growth of the resource at time t is

$$\dot{S(t)} = S(t)\left[g\left(1 \Leftrightarrow \frac{S(t)}{K}\right) \Leftrightarrow L_H(t)\beta E(t)\right] \tag{11}$$

in which $\dot{S(t)} = \frac{dS(t)}{dt}$ etc.

The differential equation governing the motion of E is derived directly from differentiation of (5) with respect to t :

$$\dot{E(t)} = H(t) + M(t) \Leftrightarrow \delta E(t)\,,$$

which can be written equivalently as

$$\dot{E(t)} = E(t)\left[\beta\{1 + L_H(t)[S(t) \Leftrightarrow 1]\} \Leftrightarrow \delta\right]\,. \tag{12}$$

Note that equations (11) and (12) do not fully define the dynamics of the state variables E and S, since $L_H(t)$ is not predetermined at t. As is shown in the next section, the equilibrium value of $L_H(t)$ is easily found in autarky.

3 Autarky

This section is concerned with the evolution of the human capital and resource stocks, and of welfare, in autarky over time. Analysis begins with the momentary equilibrium and the equations governing the dynamic motion of S and E, all of which are straightforward in the closed economy case. The remainder of this section is devoted to discussion of growth and welfare when the conditions for an interior steady state are, and are not, met. In particular, the absence of outcomes in which economic growth is sustained is emphasized.

3.1 Momentary Equilibrium

Because preferences require that each good be consumed if any are consumed, in autarky the economy must be diversified with $L_H(t) \in (0,1)$ for all t during autarky. This requires that $p(t) = \frac{1}{S(t)}$ and thus income $I(t)$ is simply $\beta E(t)$. Substituting this into the Marshallian demand for manufactures (equation (3)) and setting demand equal to supply (equation (8)) reveals a simple condition for the market clearance in the market for M:

$$L_H(t) = a \qquad \forall t \, .$$

That is, in autarky the allocation of labor to sectors at time t is independent of prices, and hence independent of $S(t)$ and $E(t)$. The instantaneous indirect utility function is also quite simple and can be rewritten as a function of $S(t)$ and $E(t)$ alone:

$$\tilde{V}(S(t), E(t)) = \eta \beta E(t) S(t)^a \, . \tag{13}$$

3.2 Dynamics in Autarky

Since the allocation of labor to the two sectors in autarky is constant, the percentage rates of change of S and E at t form a system of first order non-linear differential equations defined over the space $0 \leq S \leq K$, $0 \leq E$:

$$\dot{S(t)} = S(t) \left[g \left[1 \Leftrightarrow \frac{S(t)}{K} \right] \Leftrightarrow a \beta E(t) \right] \tag{14}$$

and

$$\dot{E(t)} = E(t) \left[\beta [1 + a[S(t) \Leftrightarrow 1]] \Leftrightarrow \delta \right] \, . \tag{15}$$

The reader may recognize that together equations (14) and (15) form a predator-prey model, in which the resource is the prey, while technology is the predator.[5]

The phase diagram representing the solution to these two differential equations is graphed in Figure 1.

This graph is drawn for the case where the steady state is a spiral point (see footnote 6) and

$$(1 \Leftrightarrow a)\beta < \delta < (1 \Leftrightarrow a)\beta + a\beta K\,; \tag{16}$$

condition (16) is a necessary and sufficient condition for the existence of a strictly

[5] Brander and Taylor (Forthcoming(b)), allowing the labor force rather than technology to vary over time, also find a predator prey relationship between the environment and other productive factors in a closed economy. This should not be surprising since both population growth and technological improvement increase the *effective* workforce, which is what ultimately enters into the production functions for H and M. This equivalence implies that controlling population growth without restricting effective harvesting effort is not sufficient to prevent environmental deterioration.

interior steady state in S and E. Denote the interior steady state values of S and E in autarky by $S^{SS,aut}$ and $E^{SS,aut}$, which are found by setting $\dot{S(t)} = \dot{E(t)} = 0$ in (14) and (15), assuming that $S(t) > 0$, $E(t) > 0$, and solving:

$$S^{SS,aut} = \frac{\frac{\delta}{\beta} \Leftrightarrow (1 \Leftrightarrow a)}{a} \tag{17}$$

$$E^{SS,aut} = \frac{g}{a\beta}\left(1 \Leftrightarrow \frac{\frac{\delta}{\beta} \Leftrightarrow (1 \Leftrightarrow a)}{aK}\right) . \tag{18}$$

Note that, when $S^{SS,aut}$ is interior, $S^{SS,aut}$ and $E^{SS,aut}$ form an asymptotically stable steady state[6] of the system formed by equations (14) and (15).

Lastly, note that the percentage rate of change of instantaneous utility $\frac{\dot{V(t)}}{V(t)}$ is simply a linear combination of the percentage rates of change of E and S:

$$\frac{\dot{V(t)}}{V(t)} = \frac{\dot{E(t)}}{E(t)} + a\frac{\dot{S(t)}}{S(t)} \tag{19}$$

which, by manipulation of equations (11) and (12), equals

$$g + [1 \Leftrightarrow a]\beta \Leftrightarrow \delta + S(t)\left[a \Leftrightarrow \frac{g}{K}\right] \Leftrightarrow a\beta E(t) . \tag{20}$$

This implies that, since E and S can both cycle, so too can welfare in the economy. From the counterclockwise path of S and E in Figure 1, one can see that the autarkic

[6] If δ is less than or greater than

$$(1 \Leftrightarrow a)\beta + \frac{a\beta K}{1 + \frac{g}{4a\beta K}} ,$$

then $S^{SS,aut}$ and $E^{SS,aut}$ form an asymptotically stable spiral point or improper node, respectively, of the system defined by (14) and (15). If instead

$$\delta = (1 \Leftrightarrow a)\beta + \frac{a\beta K}{1 + \frac{g}{4a\beta K}} ,$$

then $(S^{SS,aut}, E^{SS,aut})$ may form either an asymptotically stable spiral point, improper node, or proper node. See Chapter 9, Boyce and DiPrima (1997).

economy can enjoy periods of prosperity via increases in the stocks of both human capital and the resource. This will be followed by a period in which instantaneous utility is still rising due to increasing human capital, but in which the resource is being depleted. This will in turn be followed by an economic downturn in which both stocks are declining. This can be seen as a sort of dark ages in which there is environmental poverty and, relative to their predecessors, workers are inept. Welfare recovery can occur however, and will be led by environmental recovery prior to any intellectual recovery.

3.3 When the Autarkic Steady State Does Not Exist

In the previous section the dynamics of S, E, and instantaneous indirect utility V were discussed under the condition that there existed an interior steady state in S and E. However the two cases that violate condition (16) are equally interesting: in each, continually declining welfare is inevitable because of either resource extinction or declining human capital.

Consider the case where the rate of depreciation is small, i.e. $\delta < (1-a)\beta$. Under these parameter values, from (15) it is clear that for any $S \in [0, K]$, if $E > 0$ then the percentage rate of increase in human capital is positive and greater than or equal to $(1-a)\beta-\delta > 0$, and hence E grows without bound. This means that harvesting effort $a\beta E(t)$ also increases without bound. Once $a\beta E$ exceeds the intrinsic growth rate g, then resource contraction has become inevitable and S eventually falls to zero. In terms of the economy's Production Possibilities Frontier (PPF), this corresponds to a progressive decrease in the H intercept, and a continual increase in the M intercept; that is, in H, M space the PPF contracts inwards to approach a vertical line at $H = 0$. This possibility is drawn in Figure 2.

Note that this contraction of the PPF to the M axis is not merely an artifact of the Cobb-Douglas preferences employed. Any time that the allocation of labor to harvesting ($\bar{L_H}$) is fixed—perhaps due to labor rigidity across sectors, or government compensation schemes—if $\beta(1 \Leftrightarrow \bar{L_H}) < \delta$, then the economy will be on a path to a contracting PPF as graphed in Figure 2.

The contraction of the PPF to the M axis has dramatic implications for the dynamic path of instantaneous utility. Since the indifference curves do not cross the axes, contraction of the PPF towards the M axis eventually translates into monotonically declining instantaneous utility. To verify this, note from (20) that, once

$$E(t) > \frac{g + \beta(1 - a) - \delta + \max\{Ka - g, 0\}}{a\beta},$$

$\frac{\dot{V}(t)}{V(t)} < 0$. That is, once E rises above some threshold, then the percentage rate of change of instantaneous utility is negative, and will continue to be negative for the remainder of the autarky regime. More generally, when the allocation of labor is fixed at some value $\bar{L}_H$ and the indifference curves do not cross the M axis, then, if $\delta < (1 - \bar{L}_H)\beta$, welfare decline is inevitable since the output of H available for consumption will decline to zero. This simple yet critical result is summarized in the following proposition:

Proposition 1 *If*

1. *the allocation of labor to harvesting is constant*
2. *the rate at which human capital depreciates is sufficiently low*
3. *the indifference curves do not cross the M axis*

then, in autarky, resource depletion and continual welfare decline are inevitable.

Returning to the specific example of Cobb-Douglas preferences, the other case in which condition (16) is violated is when $\delta > (1 - a)\beta + a\beta K$. In this case continual welfare decline is also inevitable, but not because unbounded accumulation of human capital drives the resource stock to extinction, but because the rate of depreciation is so great that the economy suffers continual technological regress regardless of how close the resource stock is to its carrying capacity.

In summary, under Cobb-Douglas preferences there exist no parametric values in which the autarkic economy achieves continued utility growth. As shown above, if the rate of human capital depreciation δ is either sufficiently high or sufficiently low, the economy will eventually enter a never ending phase of welfare decline. For intermediate values of depreciation, the growth rate of instantaneous utility in the economy will converge to zero. These results are restated below in proposition form:

Proposition 2 *When preferences are Cobb-Douglas there is no combination of parameters a, δ, β, g, K under which the autarkic economy maintains positive welfare growth indefinitely.*

This, however, need not be the case when $\beta > \delta$ and L_H is chosen by a social planner. Contained in the set of dynamic paths of S and E consistent with equations of motion (11) and (12) is a set of paths along with S and H eventually stabilize at constant values, and both E and M eventually grow without bound. Such a path is obtained when L_H is chosen to decline at a percentage rate equal to the percentage rate of growth of human capital. This decrease in L_H stabilizes the resource stock and permits continual growth of welfare via increases in M alone. However, as noted above, on its own the autarkic economy will not move to one of these paths, since the intertemporal externalities associated with open access do not give any workers unilateral incentive to exit the harvesting sector so as to maintain a stable resource stock. Contrastingly, trade may induce such a relocation of labor. The consequences of such labor relocation for growth and welfare, and the conditions under which it will occur in trade, are discussed thoroughly in section 4.

Before concluding this section, however, note that for the remainder of this paper the assumption that $\delta < \beta$ will be imposed. This restriction is made because $\beta \leq \delta$ implies that manufacturing led growth is not possible under any trade regime, an implication that is clearly at odds with development experience over the last several centuries. This assumption also simplifies the discussion of dynamics in the Small Open Economy that follows in the next section, without ruling out the possibility of a zero growth trap. For the interested reader, dynamics in a Small Open Economy in which $\beta < \delta$ are summarized in the Appendix.

4 Small Open Economy

The focus is now turned to the dynamic behavior of the resource and human capital stocks in the Small Open Economy. The emphasis throughout this section is on the factors determining whether trade will put the Small Open Economy on a path toward sustainable growth or toward a zero growth steady state. Establishment of the conditions under which steady state welfare is lower in trade than in autarky is also stressed. Analysis begins with characterization of the momentary trading equilibrium.

When the economy is trading as a Small Open Economy, the relative price of the harvested good p^T is given, and the economy need no longer be diversified. This makes the dynamics of the two state variables S and E much more complex than in autarky. In particular, since $L_H(t)$ is no longer fixed, the differential equations governing S and E are once again as given by equations (11) and (12), which are two differential equations in *three* state variables S, E and L_H. The next three paragraphs are allocated to reducing the number of state variables to *two*. This will be accomplished by noting first that the value of $L_H(t)$ must be either 0 or 1 whenever the economy has a strict momentary comparative advantage in one of the two goods;

it will then be shown that, whenever the two sectors are instead equally profitable, instantaneous adjustments in the allocation of labor will equate $L_H(t)$ with a unique, single valued function.

To begin with, note that the condition under which manufacturing offers the highest wage—namely, when the value of a worker's output in harvesting, $p^T S(t)\beta E(t)$, is less than the value of the worker's output in manufacturing, $\beta E(t)$—is equivalent to the following inequality:

$$S(t) < \tfrac{1}{p^T} \,. \tag{21}$$

If (21) holds, then all labor will be employed in the manufacturing sector, and $L_H(t)$ will equal zero. If instead

$$S(t) > \tfrac{1}{p^T} \,, \tag{22}$$

then labor will find its greatest renumeration in harvesting such that $L_H(t) = 1$.

Lastly, if $S(t) = \frac{1}{p^T}$, then the economy may diversify, with any labor allocation $L_H(t)$ in the interval $[0, 1]$ consistent with *momentary* equilibrium. However, as mentioned a stability argument can be made that $L_H(t)$ will equal a single valued function, $L^*_H(t)$, whenever $S(t) = \frac{1}{p^T}$. To find this function, set $S(t) = \frac{1}{p^T}$ and $\dot{S}(t) = 0$ in (11) and isolate the variable representing the allocation of labor to harvesting; let this be the function $L^*_H(t)$:

$$L^*_H(t) = \frac{g}{\beta E(t)}\left[1 \Leftrightarrow \frac{1}{p^T K}\right] \,. \tag{23}$$

Now suppose $S(t) = \frac{1}{p^T}$, but that $L^*_H(t) < L_H(t) \leq 1$. Then by (11), $\dot{S}(t) < 0$ and wages in manufacturing are rising at a faster rate than in harvesting: all workers want to exit harvesting and enter manufacturing, putting downward pressure on L_H.

Similarly, if harvesting employment $L_H(t)$ is instead less than $L_H^*(t) \leq 1$, then from (11), $\dot{S}(t) > 0$ and any workers in manufacturing want to change sectors in search of the faster growing wage in harvesting, putting upward pressure on L_H. Since adjustments are instantaneous, allocations of labor that do not follow $L_H^*(t)$ are instantly 'corrected', returning L_H to $L_H^*(t)$. This argument applies whenever $L_H^*(t) \leq 1$, or, equivalently, whenever

$$E(t) \geq \frac{g}{\beta}\left[1 \Leftrightarrow \frac{1}{p^T K}\right] . \tag{24}$$

If instead $L_H^*(t) > 1$, then since $L_H(t)$ can be no larger than 1, the harvesting wage must be rising faster than the manufacturing wage, and all workers will be attracted to harvesting. Hence, for the remainder of section 4, it is assumed that $L_H(t) = min\{L_H^*(t), 1\}$ whenever $S(t) = \frac{1}{p^T}$. And at this point, a digression is made to introduce one other function, $E^*(S)$, which defines the $\dot{S}(t) = 0$ isocline when $L_H = 1$:

$$E^*(S) \equiv \frac{g}{\beta}\left[1 \Leftrightarrow \frac{S}{K}\right] . \tag{25}$$

The reader will notice that, when evaluated at $S = \frac{1}{p^T}$, the function E^* also equals the right hand side of equation (24); this function will be made use of shortly.

Using the discussions in the previous three paragraphs, the differential equations governing S and E can now be written solely in terms of S and E. These are piecewise functions. However, for expositional purposes S and E space will instead be divided into four "regimes", each with a corresponding system of differential equations governing $S(t)$ and $E(t)$:

If $S(t) \in [0, \frac{1}{p^T})$ then

$$\dot{S}(t) = S(t) g \left[1 \Leftrightarrow \frac{S(t)}{K}\right] \geq 0 \tag{26}$$

$$\dot{E}(t) = E(t)(\beta \Leftrightarrow \delta) > 0\,; \tag{27}$$

If $S(t) \in (\frac{1}{p^T}, K]$ then

$$\dot{S}(t) = S(t)\left[g\left[1 \Leftrightarrow \frac{S(t)}{K}\right] \Leftrightarrow \beta E(t)\right] \tag{28}$$

$$\dot{E}(t) = E(t)\left[\beta S(t) \Leftrightarrow \delta\right]\ ; \tag{29}$$

If $S(t) = \frac{1}{p^T}$ and $E(t) \geq E^*(\frac{1}{p^T})$ then

$$\dot{S}(t) = 0 \tag{30}$$

$$\dot{E}(t) = E(t)[\beta \Leftrightarrow \delta] + {p^T}^2 g\left(p^T \Leftrightarrow \frac{1}{K}\right)\left(p^T \Leftrightarrow 1\right)\ ; \tag{31}$$

Otherwise

$$\dot{S}(t) = g\left[1 \Leftrightarrow \frac{S(t)}{K}\right] \Leftrightarrow \beta E(t) > 0 \tag{32}$$

$$\dot{E}(t) = \frac{\beta}{p^T} \Leftrightarrow \delta\,. \tag{33}$$

Together these sets of equations (26),(27);...;(32),(33) can be combined to construct a regime-shifting phase diagram that takes on one of two forms, depending on the magnitude of p^T. These two cases are drawn in Figures 3 and 5, and each case is discussed in turn.

4.1 Case 1: Low World Price ($p^T < \frac{\beta}{\delta}$)

With reference to welfare in the Small Open Economy, the first case to be considered is the most optimistic, since growth is shown below to be sustainable and inevitable in trade when $p^T < \frac{\beta}{\delta}$. This is true regardless of the values of S and E at the moment of trade liberalization, so long as each is positive. This first case is also the easiest to analyze, largely because the values of S associated with the $\dot{E}(t) = 0$ isoclines for each regime lie outside of their respective supports.[7] Therefore, so long as S and E are initially positive, $\dot{E}(t) > 0$ under any regime in Case 1.

As illustrated by the arrows of motion in the phase diagram corresponding to Case 1 (see Figure 3), while the resource stock initially may rise and then fall, S will eventually reach and remain at $\frac{1}{p^T}$ (provided $\frac{1}{p^T} < K$). That is, the economy will eventually become—and remain—diversified, experiencing a stable resource stock and

[7] For example, in the regime defined by equations (27) and (28)—the regime in which the Small Open Economy is specialized in manufacturing—the associated $\dot{E}(t) = 0$ isocline is a vertical line at $S = \frac{\delta}{\beta}$. However, this lies outside of the interval over which equations (27) and (28) are defined, since p^T is assumed greater than $\frac{\beta}{\delta}$ in Case 1.

enjoying sustainable welfare growth through increases in E alone. Associated with this growth will be attendant shifts of the PPF. These shifts will be asymmetric when both S and E are changing, and symmetric when S stabilizes at $\frac{1}{pT}$ and the economy becomes diversified, as in the scenario depicted in Figure 4.

Figure 4 also depicts the production points for the Small Open Economy. Noticeably, once S remains fixed at $\frac{1}{pT}$, the rate of harvest similarly becomes pegged at the rate of natural resource growth $G(\frac{1}{pT})$. This implies that diversified growth will be accompanied by contracting harvesting sector employment. To see this, recall from (23) that $L_H^*(t)$ and $E(t)$ are inversely related: when diversified, increases in the effectiveness of a worker in harvesting are exactly offset by reductions in harvesting employment. Moreover, once diversified, the percentage growth rate of GNP will approach a positive constant $\beta \Leftrightarrow \delta$ (as can be verified by taking the time derivative of the natural log of income from equation (9), evaluating at $p(t) = p^T$ and $S(t) = \frac{1}{p^T}$, substituting for $\frac{\dot{E}(t)}{E(t)}$ from equation (31), and taking the limit as E grows without bound).

The following recapitulates the findings in Case 1: so long as both stocks are positive at the moment of trade liberalization, the economy will converge to diversified, sustainable growth of GNP. Once diversified, the rate of harvest will stabilize at a constant level, and harvesting's share of both employment and GDP will eventually decline with time.

4.1.1 Welfare Comparisons

For the sake of brevity, comparisons of welfare between trade and autarky are restricted to the case where the economy is initially in an autarkic steady state. This is made quite simple since the conditions that characterize Case 1 imply that $S^{SS,aut} < \frac{1}{p^T}$ and $E^{SS,aut} > E^*(\frac{1}{p^T})$. Therefore, upon impact of trade liberalization the economy will specialize in manufacturing and both S and E will begin to rise. Once S reaches $\frac{1}{p^T}$, human capital will already be sufficiently high to allow non-trivial diversification, and the economy will embark on a road of continued diversification with positive growth.[8]

As suggested by the outward shifts of the PPF graphed in Figure 4, relative to the zero growth autarkic steady state, the trading economy gains at every step along the way. Upon impact of trade liberalization the economy gains from the increase

[8] This statement assumes that $\frac{1}{p^T} < K$. If this is not the case, then the economy never changes its production pattern; it will still enjoy continued long run positive growth of both E and V though.

in its consumption possibilities set that is afforded by access to different relative prices. At every point thereafter, the stocks of human capital and the resource are higher than their autarkic steady state values, increasing the productive power of the economy and hence the purchasing power of workers. In sum, in Case 1 the economy unambiguously gains from trade relative to the autarkic steady state.

4.2 Case 2: High World Price ($p^T > \frac{\beta}{\delta}$)

Case 2 provides a less certain outcome for the Small Open Economy. As discussed below, while it is still possible for the economy to enjoy continual increases in human capital and instantaneous utility while the economy is specialized in manufacturing, and in some cases while diversified, there also exist two steady states at which the growth rate of welfare is zero. Moreover, whether or not long run growth will be sustained in the Small Open Economy depends both on its level of resource depletion prior to trade and on world prices.

First, consider the existence of the steady states in Case 2. Because the value of S associated with the $\dot{E}(t) = 0$ isocline for the $L_H(t) = 1$ regime is now within that regime's support, a zero growth steady state in which the economy is specialized in harvesting exists, and is locally stable. This steady state is at $(S, E, L_H) = (\frac{\delta}{\beta}, E^*(\frac{\delta}{\beta}), 1)$.[9] In the other steady state, the economy is diversified. This occurs at

[9] Again, whether or not this steady state is a spiral point or an improper node depends on the size of δ: if

$$\delta > \frac{\beta K}{1 + \frac{g}{4BK}}$$

then $(\frac{\delta}{\beta}, E^*(\frac{\delta}{\beta}), 1)$ forms an improper node. If the inequality is reverse, then it is a spiral point. If there is equality, then this steady state is either an improper node, a

$(S, E, L_H) = (\frac{1}{p^T}, E^{SS,div}, L_H^{SS,div})$, where

$$L_H^{SS,div} = \frac{\left(\frac{\delta}{\beta} \Leftrightarrow 1\right)}{\left(\frac{1}{p^T} \Leftrightarrow 1\right)}$$

and $E^{SS,div} = \frac{1}{L_H^{SS,div}} E^*(\frac{1}{p^T})$. However, this diversified steady state has only saddle point stability, and as depicted in Figure 5, near this steady state the saddle path leading to it has positive slope. As shown below, this saddle path provides a useful analytic tool, since the division of S and E space it provides also distinguishes starting points that lead to sustained growth from those that lead to stagnation.

Specifically, consider starting values (S_0, E_0) to the northwest of the saddle path leading to the diversified steady state. Here both S and E are rising, and the economy eventually ends up diversified and the growth rate of the human capital stock (and instantaneous utility) approaches $\beta \Leftrightarrow \delta > 0$ as t goes to ∞. And, as in Case 1, as

spiral point, or a proper node. In any case, the steady state is asymptotically stable.

E grows without bound, harvesting sector employment asymptotically approaches zero. Again, sustained positive growth is accompanied by declining employment in the harvesting sector.

However, for (S_0, E_0) that lie to the southeast of the saddle path leading to the diversified steady state, the economy may also diversify for a period of time, but will eventually end up specialized in harvesting, with S and E approaching $\frac{\delta}{\beta}$ and $E^*(\frac{\delta}{\beta})$ asymptotically and the growth rate of instantaneous utility approaching zero as t approaches ∞.

Which will be the outcome for an autarkic economy opening to free trade? For an autarkic economy in steady state prior to trade liberalization, the answer depends on whether or not the economy finds its momentary comparative advantage in harvesting upon impact of trade liberalization. Specifically, if the resource stock is depleted in the autarkic steady state relative to $\frac{1}{p^T}$, i.e. $S^{SS,aut} < \frac{1}{p^T}$, then $E^{SS,aut} > E^{SS,div}$, such that the economy begins trading from the northwest of the saddle path and the Small Open Economy will be on a path toward diversification and sustainable welfare growth.

However, if instead the autarkic stock level is greater than the diversified stock (i.e. $S^{SS,aut} > \frac{1}{p^T}$) then $E^{SS,aut} < E^{SS,div}$ and the newly liberalized economy finds itself to the southeast of the saddle path[10]: this Small Open Economy ultimately converges to the harvesting steady state. Interestingly, the autarkic economy will

[10] Because $\frac{\delta}{\beta} < 1$ under Case 2, $S^{SS,aut}$ necessarily lies to the left of the $\dot{E}(t) = 0$ isocline, and hence in a region where the saddle path is strictly upward sloping. Thus, if $S^{SS,aut} > \frac{1}{p^T}$ and $E^{SS,aut} < E^{SS,div}$, then the economy must initially find itself to the southeast of the saddle path.

only find itself at the diversified steady state if $L_H^{SS,div} = a$, that is, if $p^{SS,aut} = p^T$ initially.

4.2.1 Welfare Comparisons

Clearly if prices are such that the economy begins trading from the northwest of the saddle path, and hence ends up with positive long run growth, then, as in Case 1, relative to the autarkic steady state this economy gains at every point along the way. In some senses, pre-trade resource depletion is therefore beneficial, since it places the economy's momentary comparative advantage in manufacturing and puts the economy on a path of sustainable growth.

If instead world prices induce the economy to initially specialize in harvesting, such that the Small Open Economy ends up at the harvesting steady state with zero growth in the long run, then welfare analysis is not so simple, since trade has two effects that work in opposite directions. First, the economy gains through access to world relative prices different from its own, since this increases the size of its consumption possibilities set (CPS) relative to its production possibilities set. But second, changes in the allocation of labor lead to changes in the steady state levels of each of the stocks, with consequences for the shape of the steady state PPF. Consider the following scenario: the economy opens to trade from a position of autarkic steady state, and finds itself to the southwest of the saddle path. Under the conditions defining Case 2, comparison of autarkic steady state stock levels with those in the harvesting steady state reveals that the steady state resource stock is raised, while the steady state human capital stock is lowered, by trade. Consequently, in this scenario trade lowers the M intercept on the economy's PPF, reduces the slope of its steady state PPF, and has an ambiguous effect on the H intercept. In total, the welfare effects of trade under this scenario are also ambiguous. While positive growth is not obtained, steady state welfare may be improved if the world price of the harvested good is high enough to compensate for any losses in the productive capacity of the economy. More formally, the condition under which trade raises steady state welfare in this scenario can be found through comparison of indirect utility evaluated at the steady states: at $\tilde{V}(S^{SS,aut}, E^{SS,aut})$ in autarky, where $\tilde{V}(S^{SS,aut}, E^{SS,aut})$ is the modified indirect utility function (13) evaluated at $S^{SS,aut}$ and $E^{SS,aut}$ from equations (17) and (18); and at $V(p^T, p^T \frac{\beta}{\delta} E^*(\frac{\beta}{\delta}))$ in the harvesting steady state, where $V(p^T, p^T \frac{\beta}{\delta} E^*(\frac{\beta}{\delta}))$ is the indirect utility function (4) evaluated at relative price p^T and income $p^T \frac{\beta}{\delta} E^*(\frac{\beta}{\delta})$. Substitution of terms yields

$$\tilde{V}(S^{SS,aut}, E^{SS,aut}) \gtrless V(p^T, p^T \tfrac{\beta}{\delta} E^*(\tfrac{\beta}{\delta})) \tag{34}$$

$$\Leftrightarrow \quad \frac{S^{SS,aut} E^{SS,aut}}{\frac{\delta}{\beta} E^*(\frac{\delta}{\beta})} \gtrless \left[\frac{p^T}{p^{SS,aut}}\right]^{1-a} \tag{35}$$

$$\Leftrightarrow \quad \left[\left[\frac{1+aK}{1+a}\right]\left[\frac{\beta}{\delta}\right]^2 \left[\frac{\frac{\delta}{\beta} \Leftrightarrow (1 \Leftrightarrow a)}{a}\right]^a\right]^{\frac{a}{1-a}} \gtrless p^T \tag{36}$$

where double sided arrows indicate equivalence. That is, *ceteris paribus* if p^T is sufficiently large, then steady state welfare is higher with trade that autarky, since the high world relative price compensates for the change in the economy's productive stocks. However, for intermediate values of the world price, the alteration in the Small Open Economy's production possibilities frontier, plus the increased cost of the harvested good to home consumers, render steady state welfare lower in trade than autarky.

The product of the discussion of Case 2 can be summarized in the following two propositions. The first reiterates the importance of initial conditions for the potential of long run growth in the Small Open Economy; the second dictates the growth path and/or welfare properties of the steady state that will be reached if the Small Open Economy embarked on free trade from an autarkic steady state.

Proposition 3 *When $p^T > \frac{\beta}{\delta}$, the lower is an economy's stock of human capital at the moment of trade liberalization, the lower must be the economy's resource stock if the economy is to find itself on a path toward sustainable growth.*

Proposition 4 *If $p^T > \frac{\beta}{\delta}$, and if the Small Open Economy is at autarkic steady state prior to trade liberalization, then*

1. *the economy will enjoy sustained human capital and utility increase if and only if*
$$\frac{a}{\frac{\delta}{\beta} \Leftrightarrow (1 \Leftrightarrow a)} > p^T \ ;$$

2. *the economy will end up specialized in harvesting in a zero growth steady state, with steady state welfare that is lower in trade than it was in autarky, if and only if*
$$\frac{a}{\frac{\delta}{\beta} \Leftrightarrow (1 \Leftrightarrow a)} < p^T < \left[\left[\frac{1 + aK}{1 + a} \right] \left[\frac{\beta}{\delta} \right]^2 \left[\frac{\frac{\delta}{\beta} \Leftrightarrow (1 \Leftrightarrow a)}{a} \right]^a \right]^{\frac{a}{1-a}} \ ;$$

3. *the economy will end up specialized in harvesting in a zero growth steady state, with steady state welfare that is higher in trade than it was in autarky, if and only if*
$$\frac{a}{\frac{\delta}{\beta} \Leftrightarrow (1 \Leftrightarrow a)} < p^T$$
and
$$p^T > \left[\left[\frac{1 + aK}{1 + a} \right] \left[\frac{\beta}{\delta} \right]^2 \left[\frac{\frac{\delta}{\beta} \Leftrightarrow (1 \Leftrightarrow a)}{a} \right]^a \right]^{\frac{a}{1-a}} \ .$$

5 Trade and Labor Policy

This section provides a discussion of how two very simple policies—production and employment taxes and subsidies—can affect long run growth. Consider first the impact of a tax on the production of the harvested good, in particular in the case where the Small Open Economy finds itself at (S_0, E_0) to the southeast of the saddle path in Case 2. In this case, any production tax that reduces the stock level associated with diversification, $S = \frac{1}{p^T - tax}$, to below the stock level associated with the $\dot{E}(t) = 0$ isocline when $L_H = 1$ will induce labor to specialize in manufacturing. This will permit growth of both the human capital and resource stocks. Once S and E are increased to a point above/to the northwest of the saddle path associated with no tax, then the tax can be removed and the economy will converge toward diversified sustainable growth. Basically, the production tax permits accumulation of human capital at lower stock levels than would be permitted under free trade, because the tax prevents labor from flocking to the sector with high current profits but low current contribution to accumulated human capital.

Alternatively, consider the consequences of maintaining a fixed population of workers in the harvesting sector, as would occur under a policy to tax(subsidize) all workers and provide a subsidy(tax) to $\bar{L}_H$ workers in the harvesting sector such that net wages are equalized across sectors. This would render the free trade dynamics of S and E equivalent to those in autarky: it is possible in this scenario that the economy either spirals towards negative growth (if $\delta < \beta(1 - \bar{L}_H)$), or remains trapped in a zero growth equilibrium, even when sustained growth is possible in trade via zero or contracting harvesting employment. The policy recommendation behind this simple example is straightforward: if upon trade liberalization an economy's harvesting sector is uncompetitive, subsidies aimed at maintaining traditional workforce populations in that sector may prevent long run growth from becoming sustainable.

6 Caveats

6.1 What if the elasticity of substitution is not 1?

As noted, using Cobb-Douglas preferences greatly simplifies some of the autarkic analysis and welfare comparisons. In this section robustness of the model's results to variations in the formulation of preferences is discussed. First, note that under general CES preferences the autarkic allocation of labor to harvesting is not independent of the price of the resource. For example, if the elasticity of substitution between H and M is less than 1, then $\frac{dL_H}{dS} < 0$, such that as the resource nears extinction, the allocation of labor to harvesting *increases*. In this case, so long as there is some positive rate of depreciation of human capital, there exists some interior steady state with zero growth, because the low level of output produced in the sector with most of the workers will keep E correspondingly low. However, in this case the $\dot{S(t)} = 0$ isocline is no longer linear, and local stability of the interior steady state is no longer assured.

Contrastingly, in the CES case where the elasticity of substitution is greater than 1, but finite, then $\frac{dL_H}{dS} > 0$ such that labor exits the harvesting sector when the resource stock is low. However, positive long run growth in autarky is still not assured. Consider the polar case in which H and M are perfect substitutes in consumption (for example $U(H, M) = M + bH$). Recognize that in the perfect substitutes case the phase diagram for the autarkic economy exhibits regime switching similar to that for the Small Open Economy, except that the resource level at which the economy is diversified is now at $\frac{1}{b}$ instead of $\frac{1}{p^T}$. Again, so long as $\frac{1}{b} < \frac{\delta}{\beta}$ then a zero growth steady state exists, and may occur even though long run growth via manufactures production and consumption is possible. As in Case 2 for the Small Open Economy, initial conditions will determine where the economy ends up.

Interestingly, when H and M are perfect substitutes, it is also possible for an autarkic economy that is on a path of sustainable growth to be driven into a zero growth trap if it opens too early to free trade with a world in which the price of the harvested good is greater than its autarkic price. Such a possibility is sketched in Figure 6.

6.2 Spillover of Learning by Doing Across Sectors

Throughout the analyses of sections 2, 3 and 4, it has been assumed that the effectiveness of labor increases with cumulative production, no matter what sector does the production. This assumes complete generality of human capital: there is no sector specificity of learning by doing. There are two reasons for having imposed this assumption. First, if human capital is sector specific, then *any* economy that is diversified will grow slower than one that is specialized, other things being equal. If one wants to compare growth and resource depletion in an open and specialized economy with that in an autarkic economy, then for the sake of clarity general learning by doing is more appropriate. But there is also an empirical reason to model human capital as spilling across sectors: many resource-based industries employ a high level of technological sophistication in both extraction and processing, for example sonars employed on modern fishing vessels, or image recognition technology designed for the mechanization of sawmills. It is not unreasonable to argue that such advances in the technology developed for use in resource based industries can also be be made useful elsewhere in the economy. However, for the interested reader, two cases in which human capital is sector specific are analyzed below; in the interest of brevity, the mathematics underlying this discussion are suppressed.

Consider the case implicitly assumed in Matsuyama (1992) and Sachs and Warner (1995), in which harvesting neither contributes to, nor benefits from, the accumulation of human capital. In the context of the model presented in sections 2 through 4, it can be shown that positive economic growth is sustainable in autarky (if the rate of human capital depreciation is small relative to the income share for manufactures) and in trade. However, there also exists a zero growth steady state in trade, at which the economy is specialized in harvesting. And unlike the case with general human capital, for any prices, *any* positive initial resource stock level will put the economy on a path to this zero growth steady state, if the corresponding initial human capital stock is also low.

Alternately, when harvesting is instead the sole contributor to and beneficiary of human capital accumulation, positive utility growth can be shown to be unsustainable in either autarky or trade. This is a direct result of the biological constraints of the natural resource, and the assumption that learning by doing affects primary, not value added, production. The validity of this last assumption is the subject of the next section.

6.3 What do we learn to do when we learn by doing?

One of the most dramatic implications of the model presented above is the result that welfare growth is unsustainable when the economy is either specialized in harvesting or harvesting employment is not declining. This result relies on the manner in which learning by doing affects the harvesting industry. As modeled, *ceteris paribus*, technological advance raises the rate of harvest, which decreases the sustainable level of the resource stock. By indirectly causing depletion of the resource, enough technological advance eventually leads to a decrease in overall productivity. However, if technological advance instead increases the intrinsic growth rate of the resource g, or only increases the *value* of the harvested good, then sustained growth is indeed possible with a fixed level of employment in the harvesting sector. Consider each of these possibilities.

Examples of changes in the regenerative power of the environment, the coefficient

g in this model, the regenerative power of the environment include the modern fish farm, with its use of antibiotics that arguably raise the rate at which a body of water can produce fish[11]. Similarly fertilizer on fields effectively increases the natural rate of resource growth in agriculture. However, if the resource in question is open access, then *given the choice* (which I do not give to agents in my model), no individual has an incentive to devote their learning towards increasing the resource's intrinsic growth rate when they could instead be increasing their own rate of capture.

If instead learning by doing improves the *quality* of goods, but not the rate at which they are harvested/produced, then, again, welfare growth could be sustained without requiring declining harvesting employment. However, any change in prices (as would occur with trade liberalization) that leads to an increase in harvesting employment would still lead to a decrease in the steady state resource level, thereby lowering the growth rate of the economy.

7 Conclusions

The principal goal behind this paper has been to see if there is an inherent characteristic of natural resources that can explain why so many countries and regions exporting primary goods have not been able to achieve sustained growth in real incomes. I have looked at only one aspect of this problem—technological growth in the presence of an open access renewable resource—and found stark results. First,

[11] Arguments have also been made that fish farms will ultimately decrease the regenerative rate of nearby water bodies, by providing breeding grounds for so called super-bugs.

recalling Propositions 1 and 2, under certain preferences positive welfare growth will not be sustained in the closed system (autarky). Second, trade with a world that offers a set of fixed prices can offer a remedy. But this remedy exists only so long as the rest of the world either does not value the resource highly, or, if it does, it pays handsomely for it. If instead the world values the good more, but not much more, than the autarkic steady state economy, then the country is made worse off by trade because the price rise is not sufficient to compensate for the contraction of the economy's PPF. Third, positive welfare growth can be sustained only if successive improvements in technology are accompanied by successive contractions of the labor force involved in harvesting: as we get better at hewing wood and drawing water, there should be fewer of us doing it.

Appendix

In this appendix the dynamics of S and E in the Small Open Economy are discussed for the case where δ is greater than β. Observe at the outset that sustained growth is impossible under this scenario: the best the trading economy can do in the long run when $\delta > \beta$ is maintain zero growth.

Consider first the case where p^T is low, specifically where $p^T < \frac{\beta}{\delta}$. Then there exists a single steady state

$$(S, E, L_H) = (\tfrac{1}{p^T}, E^{SS,div}, L_H^{SS,div})$$

where $L_H^{SS,div}$ and $E^{SS,div}$ are as defined on page 24. This steady state is locally stable and at it the economy is diversified. As in Case 2 the economy's pattern of specialization upon trade liberalization is not determined by the defining parameter restrictions, even if the autarkic economy was in a steady state. Interestingly, if $p^T < p^{SS,aut}$ and $a > L_H^{SS,div}$, such that in the diversified steady state the country imports good H, then the steady state values of both S and E are higher with trade than autarky and, while not enjoying constant growth as was possible in Cases 1 and 2, steady state welfare is nonetheless higher in free trade than in autarky.

But if instead $p^T > p^{SS,aut}$ while $a > L_H^{SS,div}$ then both S and E are lower in the trading steady state than in autarky, as is steady state welfare. To see this graphically, view Figure 7, which depicts the economy's steady state PPF upon impact of trade liberalization and in the trading steady state.

Since both S and E fall, there is a wholesale, but asymmetric, shift in of the economy's PPF. Since, when diversified, the PPF also gives the outer boundary of the economy's Consumption Possibilities Set, steady state instantaneous welfare is necessarily lower in trade than in autarky.

Lastly, suppose instead that p^T is high, such that $p^T > \frac{\beta}{\delta}$. Then there exists a single interior steady state, occuring when S and E equal $\frac{\beta}{\delta}$ and $E^*(\frac{\beta}{\delta})$, to which the Small Open Economy inevitably converges. Notably, in this case steady state values of each state variable are higher in autarky than in trade, such that both intercepts of the economy's PPF are decreased. Again, if the world price of the harvested good is very high, then the economy can gain from trade (in the steady state) even though its PPF has shifted in.

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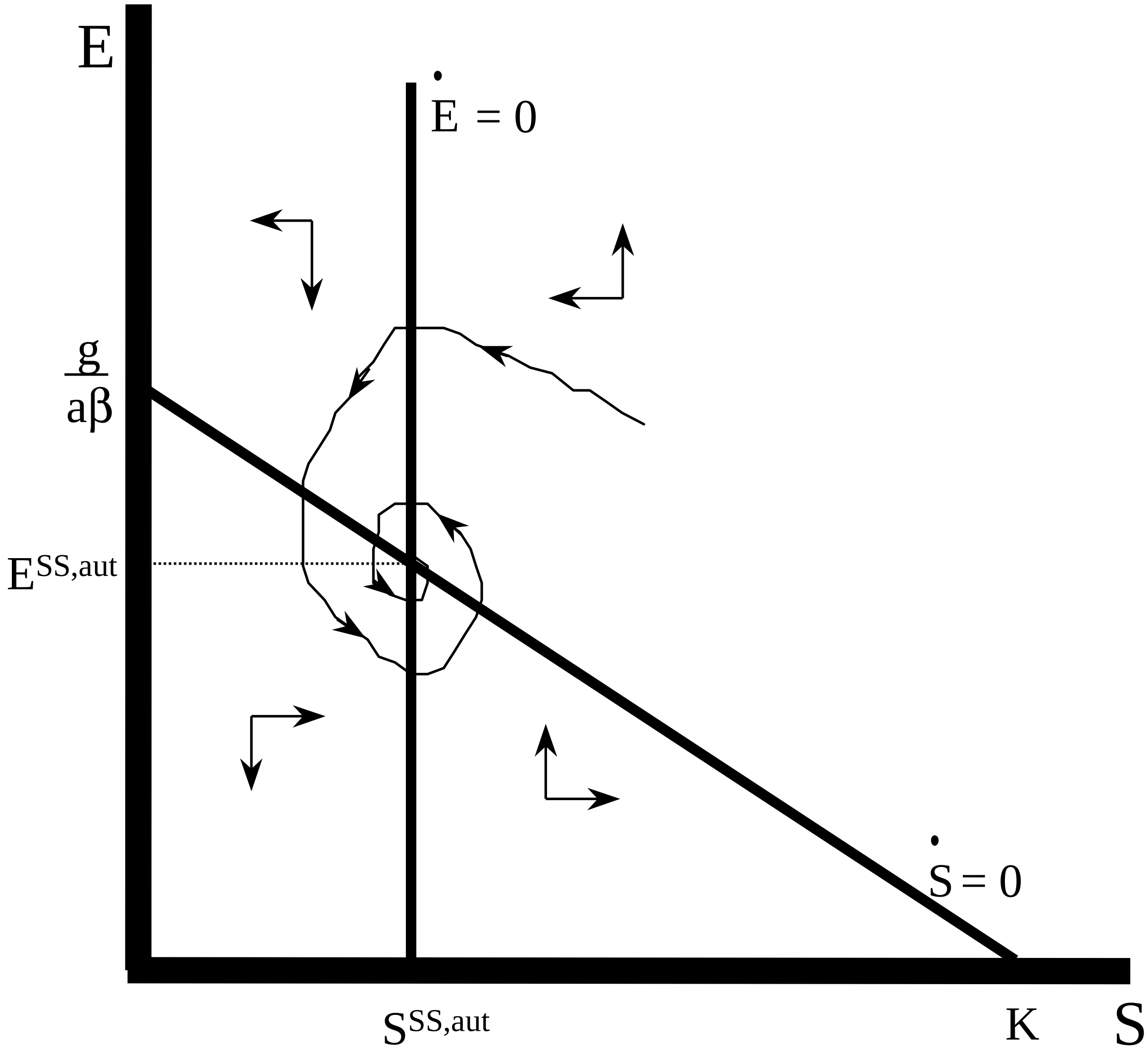

Figure 1: Phase Portrait of Resource (S) and Experience (E) in Autarky

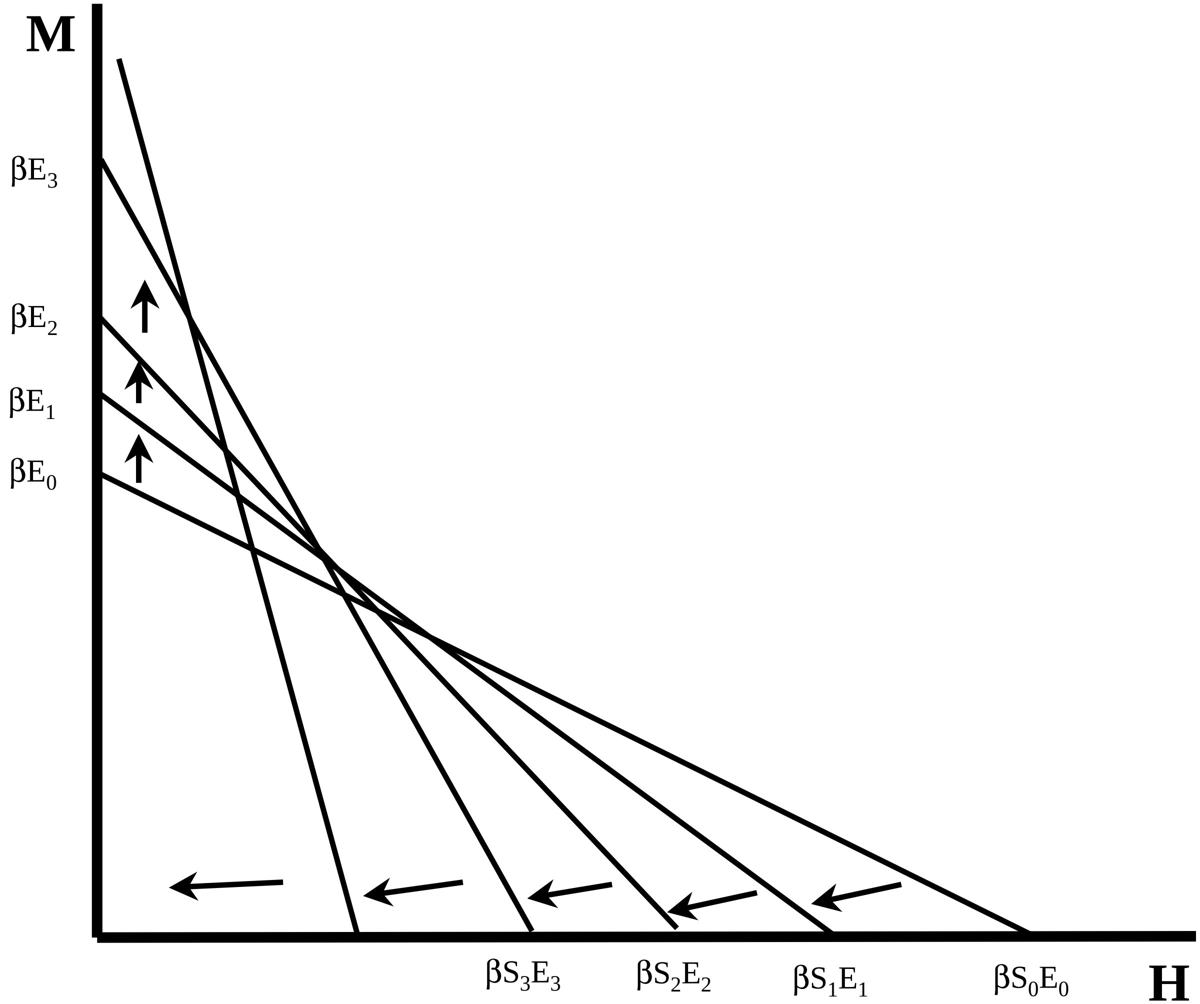

Figure 2: Autarky, $\delta<(1-a)\beta$

Fixed allocation of labor to harvesting leads to contraction of PPF toward M axis.

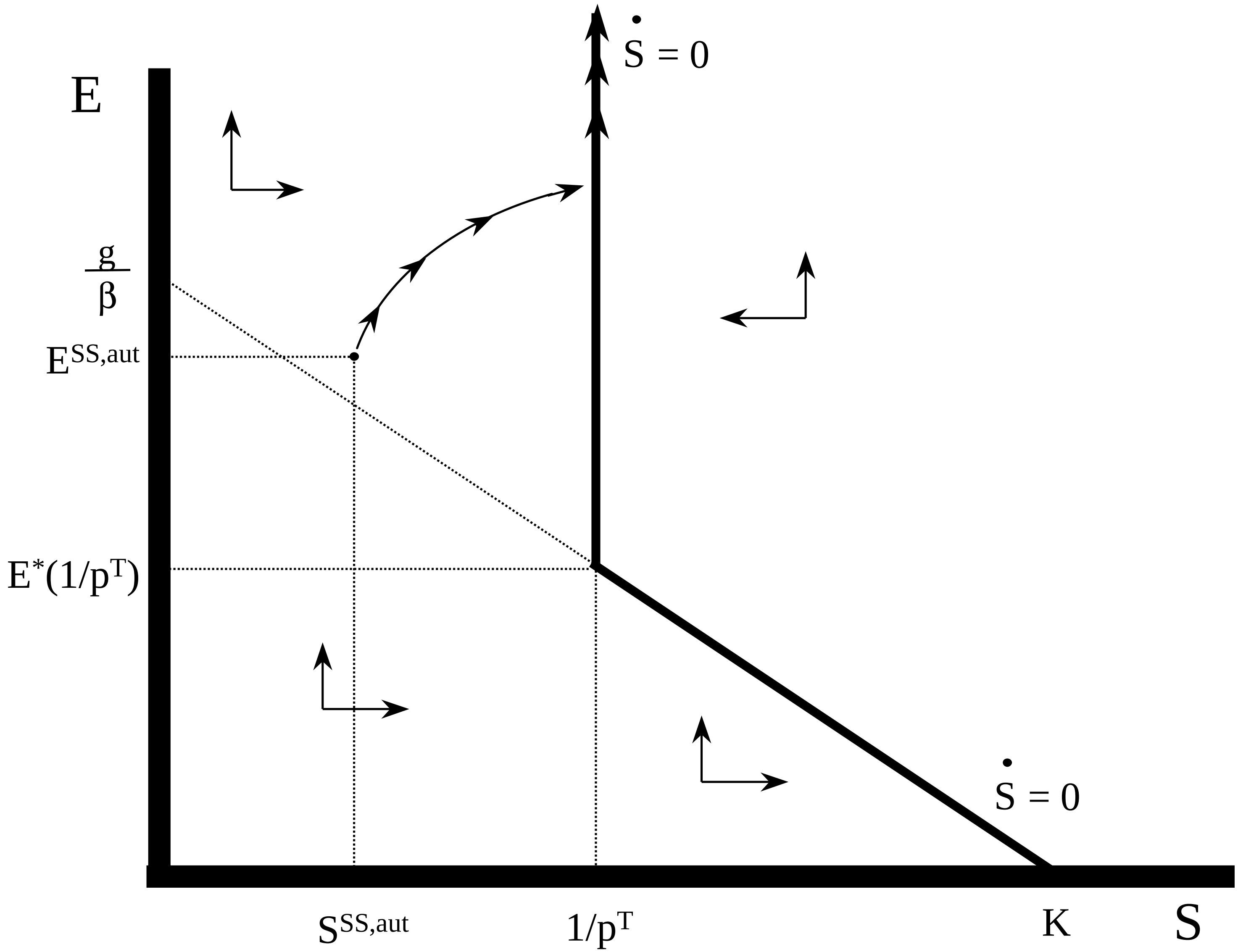

Figure 3: $\beta/\delta > p^T$;
Long Run Growth Sustainable and Inevitable

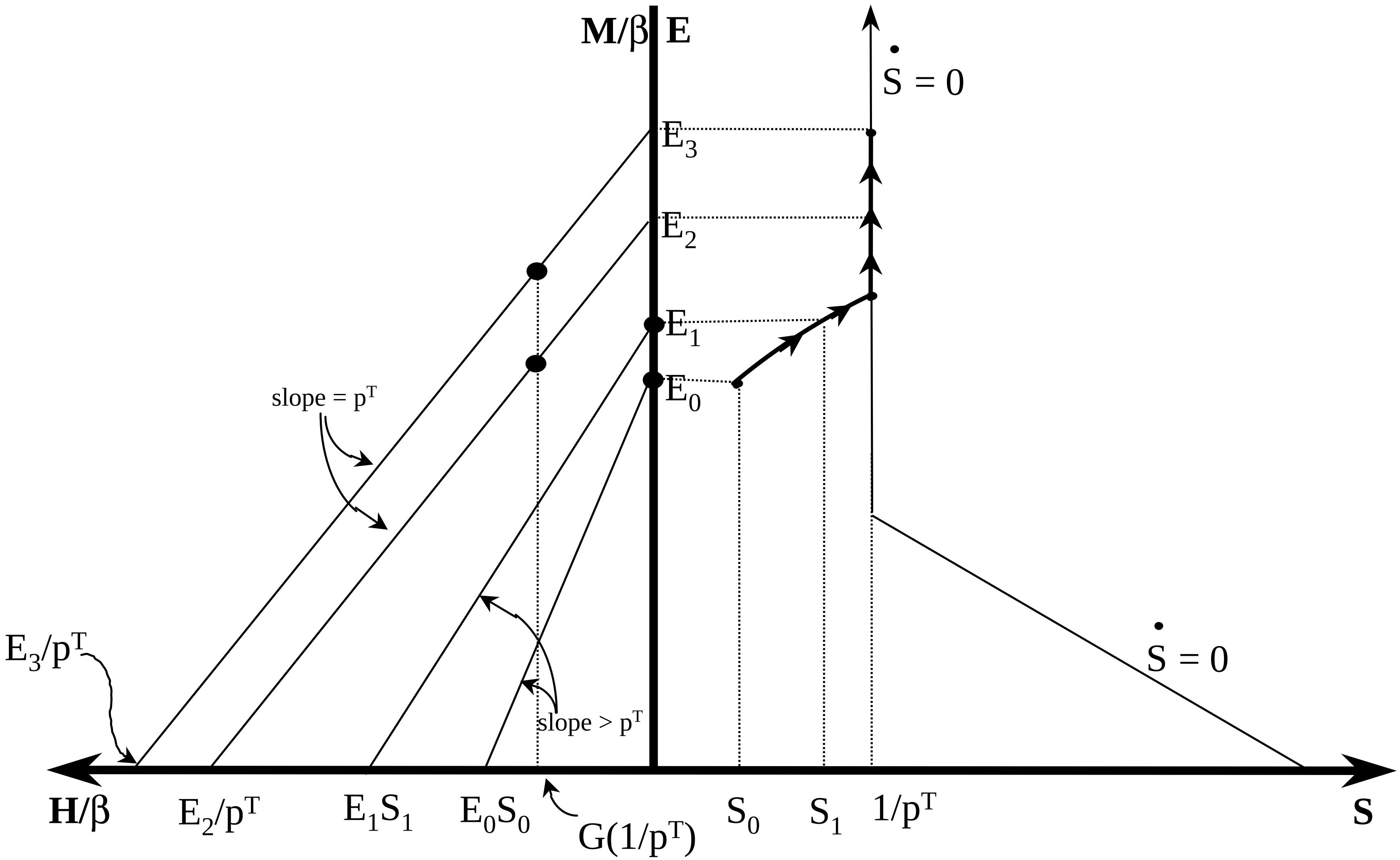

Figure 4: Case 1, production points, plus
Right Panel: Dynamic path of *S* and *E*;
Left Panel: Attendant shifts outward of the PPF.

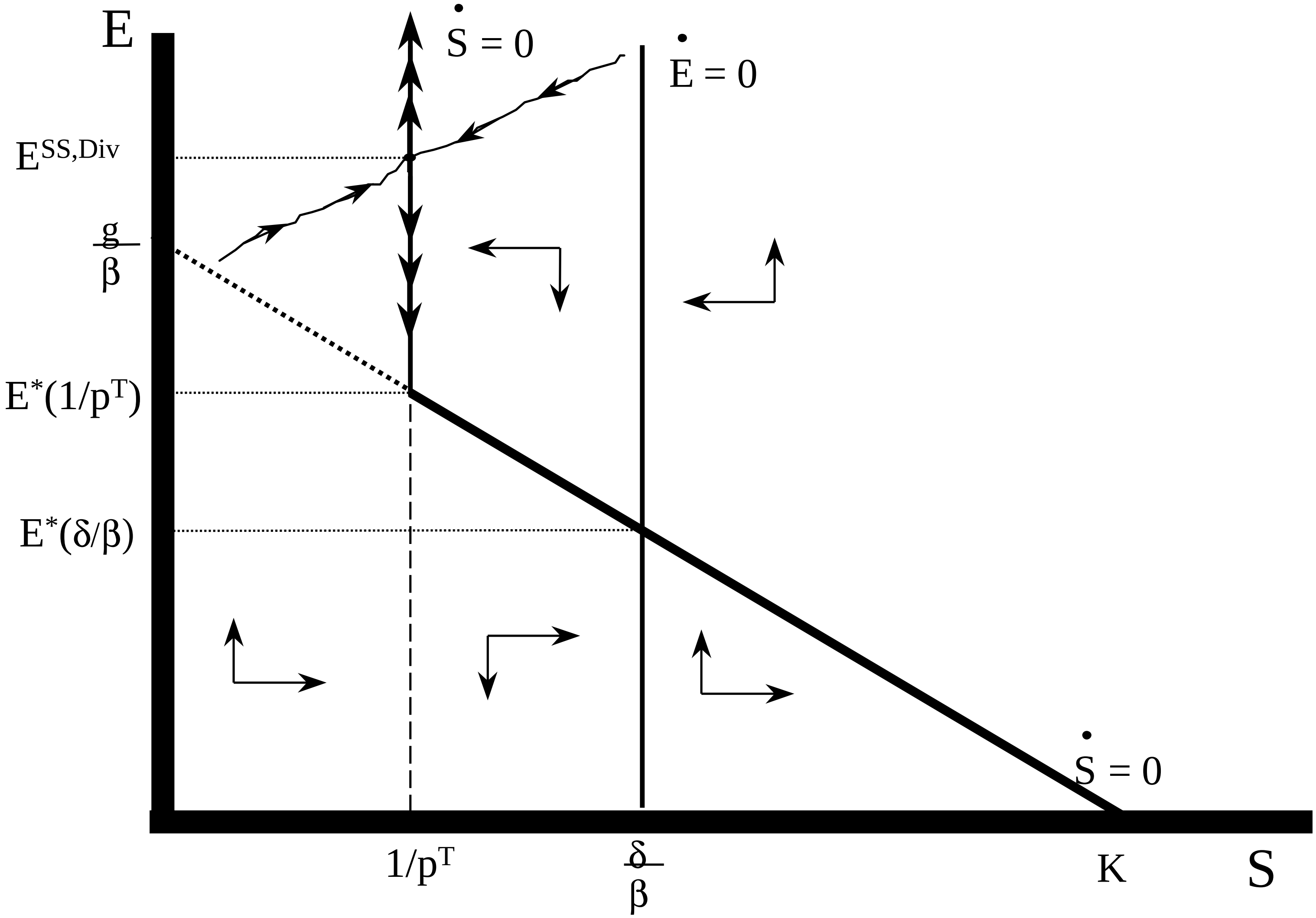

Figure 5: $\beta/\delta < p^T$;
Unstable Diversified Steady State, Long Run Growth Possible

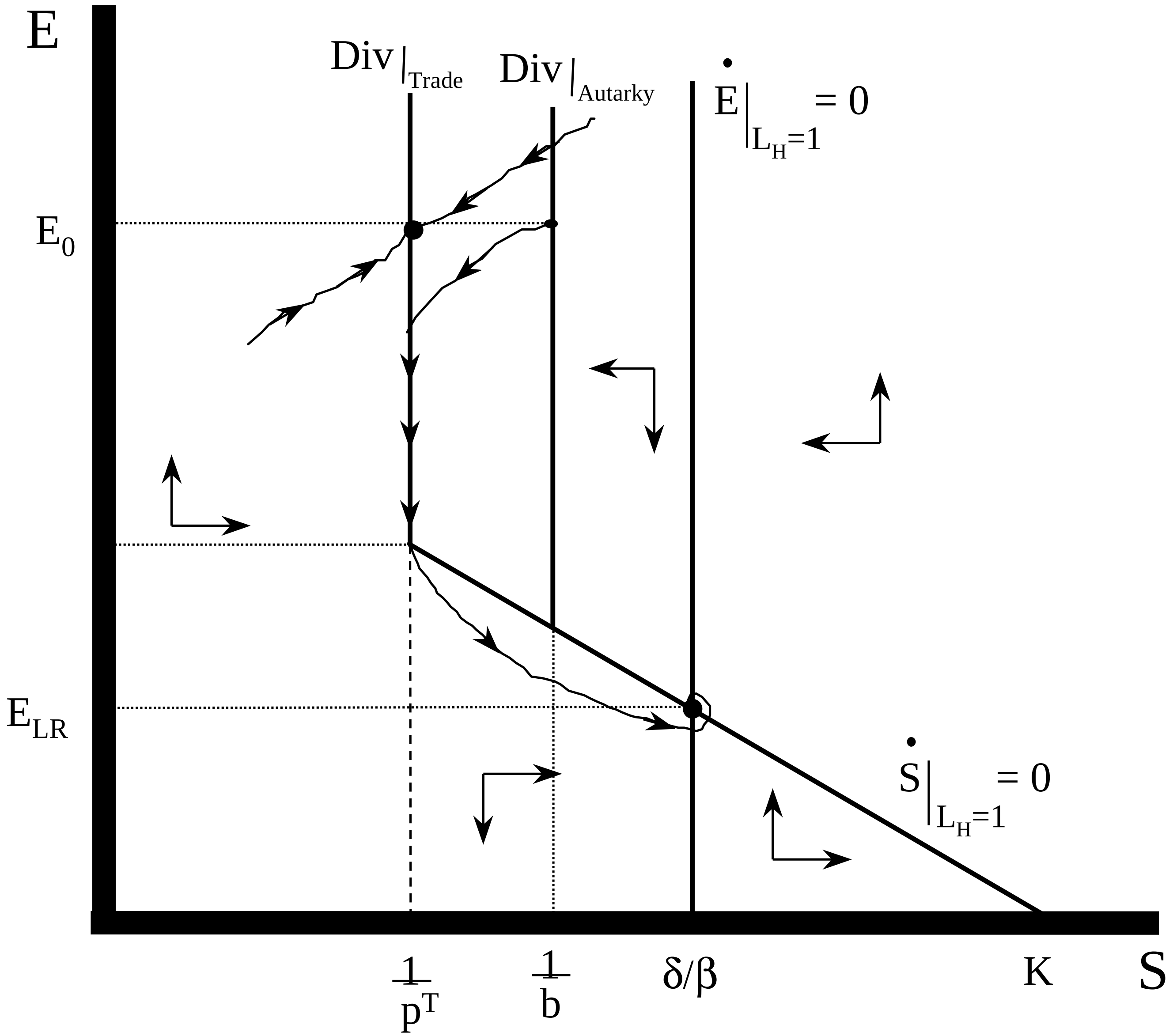

Figure 6: H and M perfect substitutes; $b<p^T$; E_0 low

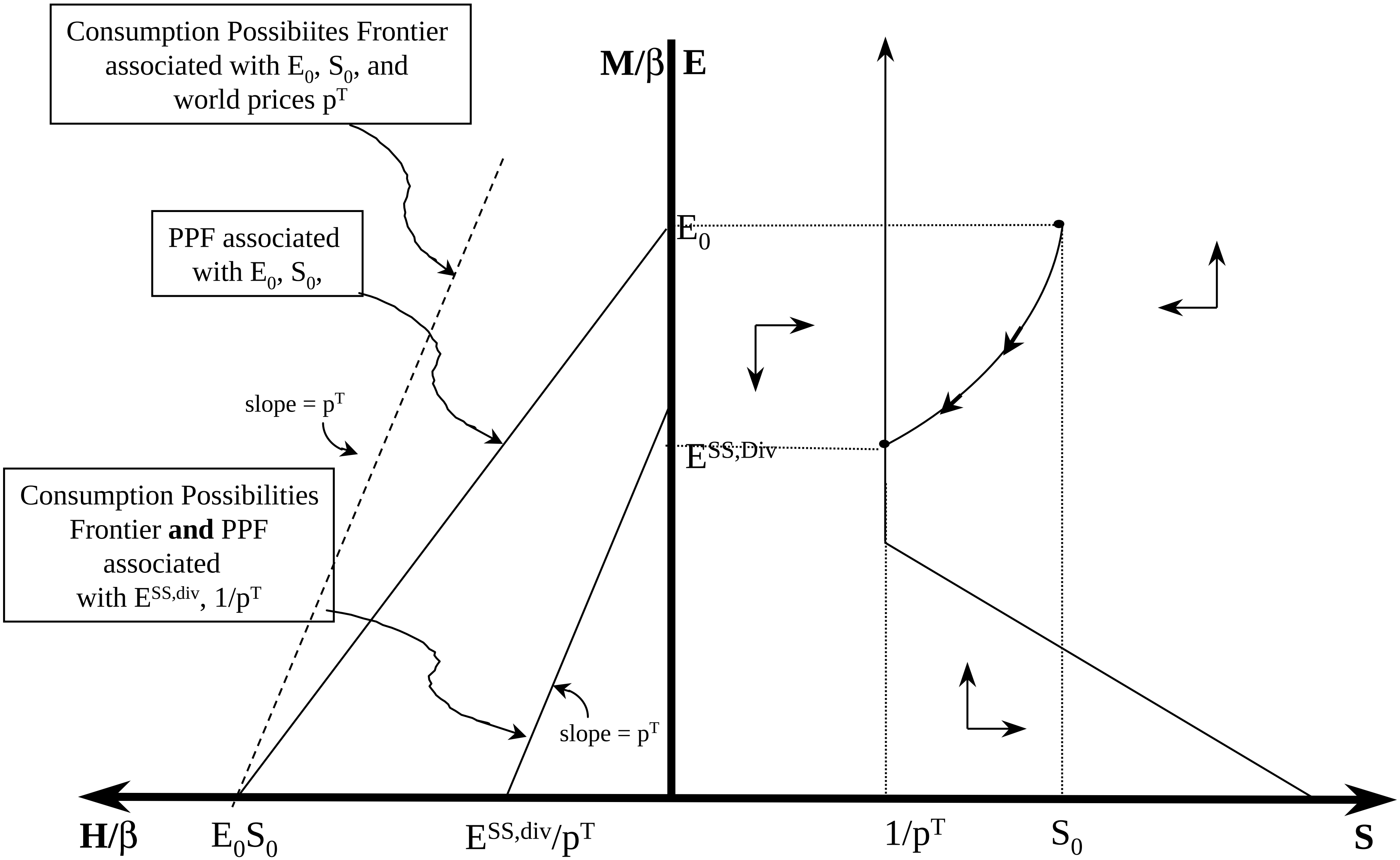

Figure 7: Welfare loss as economy moves to trade and diversification: PPF shifts inward