

# Structural Transformation in an Open Economy

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## Abstract

We study structural change in an open economy. We employ a tractable dynamic, multi-sector, two-country model to investigate the role of openness to trade and of other forces in structural transformation. A core feature of the model is an endogenous pattern of trade, dictated by comparative advantage, between and within sectors. We derive an expression linking sectoral resource allocation to expenditure shares and to the sector's net export share of total GDP. We also show that even when the elasticity of substitution is less than one, the sector with the highest productivity growth need not shrink over time; indeed it may follow the 'hump' pattern that characterizes manufacturing as a country develops.

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# 1 Introduction

In the literature on structural change, there has almost always been a strong connection between empirics and theory.<sup>1</sup> One such example is the evolution of sectoral resource allocation patterns over time. The early research by Clark (1957), Kuznets (1957, 1966), and Chenery and Syrquin (1975), among others, documented that the agriculture share of output and employment decline, while the industry and services shares of output and employment rises, as a country develops. In light of this pattern, most models of structural change developed at that time were two sector models.<sup>2</sup> However, in more recent years, Maddison (1991), Buera and Kaboski (2008), and others have shown clearly that there are three distinct sectoral allocation patterns. The new data indicate that the industry or manufacturing share of output or employment follows a “hump” pattern as a country develops, first rising, and then falling. As a consequence, three-sector models have become more prevalent in recent years. Notable examples include Kongsamut, Rebelo, and Xie (2001) and Ngai and Pissarides (2007).<sup>3</sup>

There is, however, one area in which the strong connection between empirics and theory has been largely absent. Kuznets (1967) and Chenery and Syrquin (1975) devoted considerable effort to documenting the evolving structure of international trade as a country developed. For example, they established the increasing importance of manufactured goods exports, and the declining importance of primary goods exports, as a country’s per capita income increased. More recently, there is the experience of many rapidly growing emerging market economies, such as Taiwan, South Korea, Malaysia, Thailand, Costa Rica, and Ireland. These economies opened up to international trade, leading to rising and high trade shares of GDP. They also experienced rapid structural change — indeed, most of the above economies are already past the peak of their manufacturing hump. Moreover, in a global economy, these countries affect ongoing patterns and speeds of structural transformation in advanced countries and regions such as the United States, Japan, and Western Europe.

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<sup>1</sup>From the closing years of World War II through the 1960s, Rosenstein-Rodan, Nurkse, Lewis, Hirschman, Rostow, and others developed grand theories, but these theories were informed by the careful empirical research of Clark, Kuznets, Chenery, Leontieff and others. The empirical researchers, in turn, isolated the core assertions and arguments at the heart of these theories and used them to guide their search for patterns of development.

<sup>2</sup>The sectoral divisions were often agriculture and non-agriculture, agriculture and industry (manufacturing), or capital-intensive and labor-intensive. For recent examples of these divisions, see Caselli and Coleman (2001), Laitner (2000), and Acemoglu and Guerrieri (2008). Desmet and Rossi-Hansberg (2009) present a spatial structural change model in which services and manufacturing are the sectors.

<sup>3</sup>Also, see Buera and Kaboski (2008, 2009), Foellmi and Zweimuller (2008), Rogerson (2008), Herrendorf, Rogerson and Valentinyi (2009), Restuccia, Yang, and Zhu (2008), and Verma (2008). Also, see Ju, Lin and Wang (2009) for an n-sector model of structural change. These models are all closed economy models.

In light of the older evidence and these new experiences, it is somewhat surprising that there have been virtually no models devoted to understanding the role of international trade in structural change. The goal of our paper is to address this gap and develop a model to study structural change in an open economy. Our framework is motivated by the logic that in an open economy the tight link between sectoral demand or expenditures and sectoral production that is present in a closed economy setting does not bind; sectoral demand can be met via imports and sectoral production can be exported. Hence, an open economy has immediate implications for sectoral resource allocation. Our framework is guided by a particular mechanism for how the open economy allocates sectoral resources, namely comparative advantage.

In order to clearly highlight the transmission channels involved in an open economy setting, we develop a tractable, neoclassical model with two countries and three sectors. There is one factor of production, labor. Two of the sectors are tradable (agriculture and manufacturing); the third sector is non-tradable (services). Preferences are homothetic. The motive for trade is Ricardian; productivity differences are the source of comparative advantage. Comparative advantage captures the role of the global and domestic supply-side forces, and determines intra-sector and inter-sector patterns of specialization and trade. The dynamics in the model result from sectoral productivity growth over time.

Our analysis delivers two primary results. First, we show that the employment share of either tradable sector equals the expenditure share plus net exports in that sector expressed as a share of GDP. To understand this result, consider a closed economy version of our model. Then, the employment share of manufacturing, for example, would equal the share of total of expenditures on manufacturing goods. This is simply a re-statement of the equilibrium condition that sectoral production equals sectoral spending. In an open economy, the equilibrium condition is altered by the addition of the net export term. This is a simple, but powerful, implication. It says that even if a sectoral expenditure share is declining, if its net export share is rising, then, its employment share could be rising, as well.

Second, we demonstrate how a hump in the manufacturing employment share can arise even if it is the sector with the highest productivity growth. Suppose the elasticity of substitution across sectors is less than one. Then, if a tradable sector experiences high productivity growth, its relative price and its expenditure share will tend to fall over time. In a closed economy, this would immediately imply that the employment share is also falling over time. However, in an open economy, the pattern of trade and specialization also matters. Labor will shift, in general, to the sector with the comparative advantage, and the employment share will be higher than in a closed economy to the extent the net export share is positive. Over time, if the sector's comparative advantage increases, it

will produce and export a greater fraction of output, possibly leading to an increasing employment share over time. However, this comparative advantage effect on employment cannot continue forever. At some point, the expenditure channel become dominant, and the sector's employment share will necessarily decline. We show that the two primary results hold up in more general settings, including non-homothetic preferences, non-zero trade costs, and intermediate goods.

As mentioned above, an important closed economy three-sector model is Ngai and Pissarides (2008, hereafter NP). Our model, in many respects, is an open economy version of NP. Driven by differing sectoral productivity growth rates, the NP model can generate the main facts of sectoral resource allocation over time. Under the empirically relevant elasticity of substitution of less than one, the NP model implies that the sector with the highest productivity growth will experience declining employment shares over time. This implication is counterfactual for those emerging market countries - such as South Korea - where manufacturing has been the sector with the highest productivity growth.<sup>4</sup>

Until very recently, the main contributions in open economy models of structural change were by Matsuyama. The most relevant from our perspective are Matsuyama (1992) and Matsuyama (2008). In the former paper, the core theme is that the openness of the economy can change the sign of the relationship between sectoral productivity and long run growth. However, the model is quite different from ours, as it is a small open economy with two-sectors and learning-by-doing. The latter paper is the most closely related to our model. It presents a simple Ricardian model to make the point that high manufacturing productivity growth need not lead to a decline in manufacturing employment when the country is part of a globally interdependent economy. Other recent open economy models of structural change include Galor and Mountford (2008), Stefanski (2009), and Ungor (2009).<sup>5</sup>

The rest of the paper is organized as follows. The benchmark model is presented in section 2. Section 3 briefly analyzes the autarky version of the model, and the next section presents the main derivations and discussion. We demonstrate in section 5 that the our main findings hold in more general settings including those with non-homothetic preferences, trade costs, and intermediate goods. The final section concludes.

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<sup>4</sup>If the elasticity of substitution is greater than one, then the Ngai and Pissarides model would imply that the manufacturing employment share rises monotonically over time, and that the services employment share, which tends to have the slowest productivity growth, would decline over time; both of these implications are counterfactual.

<sup>5</sup>In Stefanski (2009) and Ungor (2009), trade is motivated exogenously via Armington aggregators. Ungor studies the effect of China on structural change in the G7 countries. Galor and Mountford (2008) study the effect of trade on fertility and population growth, and on human capital acquisition.

## 2 Model

Our model builds on Eaton and Kortum (2002) and Ngai and Pissarides (2007). To highlight the role of international trade in structural change as clearly as possible, we present a simple Ricardian model with two countries and three sectors: agriculture, manufacturing and services. The agriculture and manufacturing goods are tradable and the services good is nontradable. Labor is the only factor of production, there are no intermediate goods, there are no trade costs in the tradable sectors, and preferences are homothetic. We relax each of these restrictions in section 5. Sectoral productivity grows at different rates across countries and over time; this drives the dynamics of structural change in the model.

### 2.1 Technologies

There is a single *non-tradable* good in the services sector ( $s$ ). The agriculture ( $a$ ) and manufacturing ( $m$ ) sectors each consist of a continuum of *tradable* goods along the  $[0, 1]$  interval. Each country possesses technologies for producing all the goods in all sectors. The production function for the single non-tradable good in the services sector of country  $i$  in period  $t$  is

$$Y_{ist} = A_{ist}L_{ist}, \quad (1)$$

where  $Y_{ist}$  and  $L_{ist}$  denote output and labor devoted to services, and  $A_{ist}$  denotes exogenous productivity of producing the services good.

The production function for tradable good  $z \in [0, 1]$  in sector  $q \in \{a, m\}$  of country  $i$  in period  $t$  is

$$y_{iqt}(z) = A_{iqt}(z)l_{iqt}(z), \quad (2)$$

where  $y_{iqt}(z)$  and  $l_{iqt}(z)$  denote output and labor devoted to this tradable good, and  $A_{iqt}(z)$  denotes exogenous productivity of producing this tradable good.

In a Ricardian trade model, comparative advantage is based on relative productivity differences across countries. The productivity terms,  $A_{iat}(z)$  and  $A_{imt}(z)$  determine comparative advantage. We assume that country  $i$ 's productivity in producing good  $z$  in tradable sector  $q$  and period  $t$  is the realization of a random variable  $Z_{iqt}$  drawn from the cumulative distribution function  $F_{iqt}(z) = Pr[Z_{iqt} \leq z]$ . Following Eaton and Kortum (2002), we also assume that  $F_{iqt}(z)$  is a Fréchet distribution:  $F_{iqt}(z) = e^{-T_{iqt}z^{-\theta}}$ , where  $T_{iqt} > 0$ ,  $\theta > 1$ , and  $q \in \{a, m\}$ . The parameter  $T_{iqt}$  governs the mean of the distribution; a larger  $T_{iqt}$  implies that a high efficiency draw for any good  $z$  is more likely. The larger is  $\theta$ , the lower the heterogeneity or variance of  $Z_{iqt}$ .<sup>6</sup>

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<sup>6</sup> $Z_{iqt}$  has geometric mean  $e^{\gamma/\theta}T_{iqt}^{1/\theta}$  and its log has a standard deviation  $\pi/(\theta\sqrt{6})$ , where  $\gamma$  is Euler's

## 2.2 Preferences

The representative household in country  $i$  maximizes his/her utility, which is given by

$$\sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\sigma} - 1}{1-\sigma}. \quad (3)$$

The elasticity of intertemporal substitution is  $\frac{1}{\sigma}$ .  $C_{it}$  is an aggregate of the composite agriculture  $C_{iat}$ , composite manufacturing  $C_{imt}$ , and service goods  $C_{ist}$ :

$$C_{it} = U(C_{iat}, C_{imt}, C_{ist}) = (\omega_a C_{iat}^\epsilon + \omega_m C_{imt}^\epsilon + \omega_s C_{ist}^\epsilon)^{\frac{1}{\epsilon}} \quad (4)$$

where  $\epsilon < 1$ ,  $\omega_a, \omega_m, \omega_s > 0$  and  $\omega_a + \omega_m + \omega_s = 1$ . The elasticity of substitution between sectoral goods is  $\frac{1}{1-\epsilon}$ . If  $\epsilon \in [0, 1)$ , the elasticity of substitution exceeds or equals one; that is, the sectoral goods are substitutes. If  $\epsilon < 0$ , the elasticity of substitution is less than one; that is, the sectoral goods are complements.

The composite sectoral good combines the individual tradable goods as follows:

$$C_{iqt} = \left( \int_0^1 c_{iqt}(z)^\eta dz \right)^{\frac{1}{\eta}}, \quad (5)$$

where  $c_{iqt}(z)$  is the use of good  $z$  by country  $i$  to make the composite sectoral good  $q \in \{a, m\}$  in period  $t$ , and  $\eta < 1$ . The elasticity of substitution between individual sectoral goods is  $\frac{1}{1-\eta}$ .

The household supplies the total labor endowment  $L_{it}$  inelastically and spends all labor income on consumption. The household maximizes (3), (4) and (5) subject to the following budget constraints in each period  $t$ :

$$P_{it}C_{it} = P_{iat}C_{iat} + P_{imt}C_{imt} + P_{ist}C_{ist} = w_{it}L_{it}, \quad (6)$$

$$P_{iqt}C_{iqt} = \int_0^1 p_{iqt}(z)c_{iqt}(z)dz, \quad \text{for } q \in \{a, m\}, \quad (7)$$

where  $w_{it}$ ,  $P_{it}$ ,  $P_{iat}$ ,  $P_{imt}$  and  $P_{ist}$  denote the wage rate, the aggregate consumption, agriculture composite, manufacturing composite, and services good prices, respectively, and  $p_{iqt}$  denotes the price of good  $z$  in tradable good sector  $q \in \{a, m\}$ . The budget constraints (6) and (7) ensure that balanced trade holds period-by-period.

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constant.

## 2.3 Equilibrium

All factor and goods markets are characterized by perfect competition. In particular, labor is perfectly mobile across sectors within a country, but immobile across countries. The following factor market clearing conditions hold in each period  $t$  in each country  $i$

$$L_{it} = L_{ist} + L_{imt} + L_{iat}, \quad (8)$$

where  $L_{imt} = \int_0^1 l_{imt}(z)dz$  and  $L_{iat} = \int_0^1 l_{iat}(z)dz$ .

There are no costs for trading agriculture and manufacturing goods across countries, while the services good is nontradable across countries. The following goods markets clearing conditions hold in each period  $t$  in each country  $i$ :

$$Y_{ist} = C_{ist}, \quad (9)$$

$$y_{1at}(z) + y_{2at}(z) = c_{1at}(z) + c_{2at}(z), \quad \forall z \in [0, 1], \quad (10)$$

$$y_{1mt}(z) + y_{2mt}(z) = c_{1mt}(z) + c_{2mt}(z), \quad \forall z \in [0, 1]. \quad (11)$$

Now we can define a competitive equilibrium of our model economy with country-specific and exogenous labor endowment processes  $\{L_{it}\}_{t=0}^{\infty}$  and productivity processes  $\{T_{iat}, T_{imt}, A_{ist}\}_{t=0}^{\infty}$  and structural parameters  $\{\sigma, \epsilon, \eta, \beta, \theta\}$  as follows.

A *competitive equilibrium* is a sequence of goods and factor prices  $\{p_{iat}(z), p_{imt}(z), P_{iat}, P_{imt}, P_{ist}, P_{it}, w_{it}\}_{t=0}^{\infty}$  and allocations  $\{l_{iat}(z), l_{imt}(z), L_{iat}, L_{imt}, L_{ist}, y_{iat}(z), y_{imt}(z), Y_{ist}, c_{iat}(z), c_{imt}(z), C_{iat}, C_{imt}, C_{ist}, C_{it}\}_{t=0}^{\infty}$  for  $z \in [0, 1]$  and  $i = 1, 2$ , such that given prices, the allocations solve the firms' maximization problems associated with technologies (1)-(2) and the household's maximization problem characterized by (3)-(7), and satisfy the market clearing conditions (8)-(11).

The first order optimality conditions of the household's maximization problem imply that the consumption expenditure share,  $X_{iqt}$ , in sector  $q \in \{a, m, s\}$  of country  $i$  in period  $t$  is given by

$$X_{iqt} = \omega_q^{\frac{1}{1-\epsilon}} \left( \frac{P_{iqt}}{P_{it}} \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (12)$$

where the aggregate price index  $P_{it}$  is given by

$$P_{it} = \left( \omega_a^{\frac{1}{1-\epsilon}} P_{iat}^{\frac{\epsilon}{\epsilon-1}} + \omega_m^{\frac{1}{1-\epsilon}} P_{imt}^{\frac{\epsilon}{\epsilon-1}} + \omega_s^{\frac{1}{1-\epsilon}} P_{ist}^{\frac{\epsilon}{\epsilon-1}} \right)^{\frac{\epsilon-1}{\epsilon}}. \quad (13)$$

Thus, the optimal sectoral consumption  $C_{iqt}$  is

$$C_{iqt} = \frac{X_{iqt} w_{it} L_{it}}{P_{iqt}},$$

for each sector  $q$  in country  $i$  and period  $t$ .

The price that a consumer in country  $i$  pays in period  $t$  to purchase one unit of sector  $q$  good  $z$  produced in country  $j$  is  $p_{ijqt}(z) = \frac{w_{jt}}{A_{jqt}(z)}$ . Because consumers buy goods from the country with the cheapest price, the actual price that the consumer in country  $i$  pays is given by  $p_{iqt}(z) = \min\{p_{i1qt}(z), p_{i2qt}(z)\}$ . Thus, the price of the composite good of the tradable sector  $q$  is given by

$$P_{iqt} = \left( \int_0^1 p_{iqt}(z)^{\frac{\eta}{\eta-1}} dz \right)^{\frac{\eta-1}{\eta}}. \quad (14)$$

With free trade, all tradable good prices are equalized across the two countries, and so are the prices of the tradable sector composites.

### 3 Structural Change under Autarky

We begin our analysis of the model by developing the pattern of structural change in a closed economy or under autarky.<sup>7</sup> We focus on the sectoral allocation of employment. The results developed here will allow us to highlight the contribution of international trade on structural change, which we study in the following section.

We start with sectoral prices. Under autarky, all goods are produced domestically. It is straightforward to show, for any country  $i$  and period  $t$ ,

$$P_{iat} = \frac{w_{it}}{A_{iat}}, \quad P_{imt} = \frac{w_{it}}{A_{imt}}, \quad P_{ist} = \frac{w_{it}}{A_{ist}}, \quad (15)$$

where  $A_{iat} = T_{iat}^{\frac{1}{\theta}}/\gamma$ ,  $A_{imt} = T_{imt}^{\frac{1}{\theta}}/\gamma$ , and  $\gamma = (\Gamma(1 - \frac{\eta}{1-\eta} \frac{1}{\theta}))^{\frac{\eta-1}{\eta}}$ . We need to assume  $\frac{1}{1-\eta} < 1 + \theta$  to have a well-defined price index. Under this assumption, the parameter  $\eta$ , which governs the elasticity of substitution between goods within a sector, can be ignored because it appears only in the constant term  $\gamma$ . Thus, a continuum of goods in the agriculture or manufacturing sector can be essentially reduced to one composite good with productivity processes given by  $\{A_{iqt}\}_{t=0}^{\infty}$  in sector  $q \in \{a, m\}$ .

We then study the sectoral expenditure and labor shares. In the closed economy, the

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<sup>7</sup>We use ‘autarky’ and ‘closed’ interchangeably. Autarky can be thought of as a special case of our model in which the trade costs are infinitely high. The implications are similar to those in Ngai and Pissarides (2007).

sectoral labor share equals the sectoral expenditure share under the feasibility conditions:

$$l_{igt} = \frac{L_{igt}}{L_{it}} = \frac{w_{it}L_{igt}}{w_{it}L_{it}} = \frac{P_{igt}C_{igt}}{w_{it}L_{it}} = X_{igt}, \quad (16)$$

for each sector  $q \in \{a, m, s\}$ . Thus, both the labor and expenditure shares depend on the relative prices as are described by (12) and (13). In particular, the higher is the sectoral price relative to the aggregate price, the higher the sectoral expenditure and labor shares are. Moreover, the lower is the sectoral productivity level relative to the weighted average productivity level, the higher the sectoral expenditure and labor shares are.

Now let us consider dynamics. Let  $\hat{Z}_{igt}$  denote the growth rate of variable  $Z_{igt}$ . For example,  $\hat{A}_{igt} = A_{igt}/A_{igt-1}$ . Also, let the wage rate be the numeraire, so that  $w_{it} = 1$  for any  $t$ . Then, we have, for any  $q \in \{a, m, s\}$ ,

$$\hat{l}_{igt} = \hat{X}_{igt} = \frac{\epsilon}{\epsilon - 1}(\hat{P}_{igt} - \hat{P}_{it}), \quad (17)$$

where  $\hat{P}_{it} = X_{iat}\hat{P}_{iat} + X_{imt}\hat{P}_{imt} + X_{ist}\hat{P}_{ist}$ . Thus, the elasticity of substitution links changes in sectoral labor shares  $\hat{l}_{igt}$  to changes in sectoral relative prices  $\hat{P}_{igt} - \hat{P}_{it}$ . When the elasticity of substitution between sectors is less than one, i.e.,  $\epsilon < 0$ , a sector whose relative price is growing experiences growing expenditure and labor shares. In contrast, when the elasticity is larger than one, i.e.,  $\epsilon \in (0, 1)$ , a sector whose relative price is growing experiences declining expenditure and labor shares. Finally, when the elasticity is one, i.e.,  $\epsilon = 0$ , there is no structural change: sectoral expenditure and labor shares are constant over time.

We can write the growth rate of sectoral labor shares in terms of the growth rates of sectoral productivities using (15):

$$\hat{l}_{igt} = \hat{X}_{igt} = \frac{\epsilon}{1 - \epsilon}(\hat{A}_{igt} - \hat{A}_{it}), \quad (18)$$

where the weighted average productivity growth  $\hat{A}_{it}$  equals  $X_{iat}\hat{A}_{iat} + X_{imt}\hat{A}_{imt} + X_{ist}\hat{A}_{ist}$ . When the elasticity is less than one, sectors with relatively high productivity growth experience declines in employment shares. This implies that labor moves from high productivity growth sectors to low productivity growth sectors. A necessary condition for a hump pattern in the manufacturing labor share, then, is that manufacturing can have neither the highest nor the lowest productivity growth. Manufacturing productivity growth must be below average initially and above average later on for the hump to occur.

We conclude by summarizing three key implications of the autarky model. First, the sectoral labor share equals sectoral expenditure shares. Second, structural change does

not occur when the elasticity of substitution equals one. Third, with the elasticity of substitution less than one, the sector with the highest (least) productivity growth has the fastest (slowest) rate of decline in prices and expenditure shares. Thus, labor moves from the most productive sector to the least productive sector. The opposite is true when the elasticity of substitution is greater than one.

## 4 Structural Change in Open Economy

We now analyze the patterns of structural change in an open economy. In particular, we illustrate how international trade changes the implications on sectoral relative prices, expenditure shares, and labor shares in the closed economy, given the same TFP and labor endowment processes for each sector in each country. We also study the dynamics of structure change in the open economy.

### 4.1 Impact of International Trade

We begin by defining comparative advantage. Country  $i$  has a comparative advantage in manufacturing if and only if

$$\frac{A_{imt}}{A_{jmt}} > \frac{A_{iat}}{A_{jat}}. \quad (19)$$

Clearly, if country  $i$  has a comparative advantage in manufacturing, country  $j$  has a comparative advantage in agriculture.

#### 4.1.1 Prices and Welfare

Owing to free trade, the price of each tradable good  $z$  is equated across countries; hence, the prices of the composite sectoral goods are also equated across countries:  $P_{iqt} = P_{jqt}$ . Under the Fréchet distribution, Eaton and Kortum (2002) show that the price of the composite good of the tradable sector  $q \in \{a, m\}$  can be expressed as:

$$P_{iqt} = \gamma [T_{iqt}w_{it}^{-\theta} + T_{jqt}w_{jt}^{-\theta}]^{-\frac{1}{\theta}} = \left[ \left( \frac{w_{it}}{A_{iqt}} \right)^{-\theta} + \left( \frac{w_{jt}}{A_{jqt}} \right)^{-\theta} \right]^{-\frac{1}{\theta}}. \quad (20)$$

When comparing (20) to (15), we can easily show that the prices of the composite agriculture and manufacturing goods relative to the wage rate are lower in the open economy than under autarky in both countries. The extent to which the relative tradable price in country  $i$  is lower in the open economy depends on the extent to which country  $j$ 's unit cost  $\frac{w_{jt}}{A_{jqt}}$  is lower than its unit cost  $\frac{w_{it}}{A_{iqt}}$  in producing good  $q$ .

For the nontradable services good, the price that a consumer in country  $i$  pays in period  $t$  is given by  $P_{ist} = \frac{w_{it}}{A_{ist}}$ . Thus, the price of the services good relative to the wage rate is the same in the open economy as under autarky in both countries. Therefore, the aggregate price relative to the wage rate is lower in the open economy than under autarky in both countries. This implies that the welfare is higher in the open economy than under autarky in both countries.

The manufacturing price relative to the agriculture price in country  $i$  is given by

$$\frac{P_{imt}}{P_{iat}} = \frac{A_{iat}}{A_{imt}} \left[ \frac{1 + \left( \frac{w_{jt}}{w_{it}} \frac{A_{imt}}{A_{jmt}} \right)^{-\theta}}{1 + \left( \frac{w_{jt}}{w_{it}} \frac{A_{iat}}{A_{jat}} \right)^{-\theta}} \right]^{-\frac{1}{\theta}}. \quad (21)$$

If country  $i$  has a comparative advantage in manufacturing, the term inside the brackets is less than one. Thus,  $\frac{P_{imt}}{P_{iat}} > \frac{A_{iat}}{A_{imt}}$ , where the right hand side of the inequality is the relative price in the closed economy. When the countries open up to trade, the relative tradable price  $\frac{P_{imt}}{P_{iat}}$  increases in the country with comparative advantage in manufacturing, but decreases in the country with comparative advantage in agriculture. We summarize the price and welfare implications of international trade in the following proposition.

**Proposition 1 (Prices and Welfare)** *(i) The prices of the tradable goods relative to the service good are lower in the open economy than in the closed economy for both countries. (ii) The relative price  $\frac{P_{imt}}{P_{iat}}$  is higher (lower) in the open economy than in the closed economy if country  $i$  has a comparative advantage in the manufacturing (agriculture) sector. (iii) The relative price  $\frac{P_{ist}}{P_{it}}$  is higher in the open economy for both countries. The relative price  $\frac{P_{iqt}}{P_{it}}$  is lower in the open economy if country  $i$  has comparative disadvantage in sector  $q$ . (iv) The welfare, measured by  $\frac{w_{it}}{P_{it}}$ , is higher in the open economy than under autarky in both countries.*

#### 4.1.2 Expenditure Shares and Trade Patterns

We now analyze the expenditure shares in the case that the elasticity of substitution between sectors is less than one. (Cases for higher elasticities can be derived similarly.) Since the relative services price  $\frac{P_{ist}}{P_{it}}$  is higher in the open economy in both countries, the services expenditure share is also higher in the open economy in both countries. If country  $i$  has a comparative advantage in manufacturing, its relative price of the agriculture composite  $\frac{P_{iat}}{P_{it}}$  is lower in the open economy than under autarky. Thus, its agriculture expenditure share is also lower in the open economy. We however cannot sign its manufacturing expenditure share in the open economy relative to the closed economy; the manufacturing price falls

relative to services, but rises relative to agriculture. We summarize these findings in the following proposition.

**Proposition 2 (Expenditure Shares)** *Assume  $\epsilon < 0$ . (i) In both countries, the services expenditure share  $X_{ist}$  is higher in the open economy than in the closed economy. (ii) If country  $i$  has a comparative disadvantage in tradable sector  $q$ , the expenditure share  $X_{iqt}$  is lower in the open economy than in the closed economy.*

In the open economy, tradable sector expenditures are divided between domestic goods and imported goods. Under the Fréchet distribution of productivities, the share of country  $i$ 's expenditure on sector  $q$  goods from country  $j$ ,  $\pi_{ijqt}$ , is given by

$$\pi_{ijqt} = \frac{(w_{jt}/A_{jqt})^{-\theta}}{(w_{jt}/A_{jqt})^{-\theta} + (w_{it}/A_{iqt})^{-\theta}}. \quad (22)$$

Country  $i$  spends more on imports of sector  $q$  goods to the extent that country  $j$  has a lower cost of production than country  $i$  in this sector. The import shares are also affected by the parameter  $\theta$  that governs the dispersion of the productivity draws. Under free trade, the share of country  $i$ 's expenditure on sector  $q$  goods from country  $j$  equals the share of country  $j$ 's expenditure on sector  $q$  goods from country  $j$ :  $\pi_{ijqt} = \pi_{jjqt}$ . Sectoral spending that is not on imports must be on domestic goods:  $\pi_{iiqt} = 1 - \pi_{ijqt}$ .

If country  $i$  has a comparative advantage in manufacturing, it is easy to show that  $\pi_{ijmt} < \pi_{ijat}$  and  $\pi_{iimt} > \pi_{iiat}$ . That is, the fraction of manufacturing expenditure on imported manufacturing goods will be smaller than the fraction of agriculture expenditure on imported agriculture goods in country  $i$ . Intuitively, each country will import a less fraction of their expenditure on the sector with a comparative advantage.

We now characterize the patterns of international trade, which involves both intra-sector and inter-sector trade. Country  $i$ 's exports of manufactured goods are given by  $EX_{imt} = \pi_{jimt}X_{jmt}w_{jt}L_{jt}$ . It is the product of country  $j$ 's expenditure devoted to manufactured goods,  $X_{jmt}w_{jt}L_{jt}$ , and the fraction of that expenditure that is on imports,  $\pi_{jimt}$ . Similarly, country  $i$ 's imports of manufactured goods are given by  $IM_{imt} = \pi_{ijmt}X_{imt}w_{it}L_{it}$ . Thus, country  $i$ 's net exports of manufactured goods are given by  $NX_{imt} = EX_{imt} - IM_{imt}$ . Similarly, country  $i$ 's net exports of agriculture goods is:  $NX_{iat} = EX_{iat} - IM_{iat} = \pi_{jiat}X_{jat}w_{jt}L_{jt} - \pi_{ijat}X_{iat}w_{it}L_{it}$ . The balanced trade condition implies that inter-sectoral trade has to sum to zero, i.e.,  $NX_{imt} + NX_{iat} = 0$ .

We denote the net exports ratio of sector  $q$  in country  $i$  by  $N_{iqt} = \frac{NX_{iqt}}{w_{it}L_{it}}$ . We find that the direction of the inter-sector trade is determined by comparative advantage. If country  $i$  have a comparative advantage in manufacturing, it will have a positive net export ratio

in manufacturing but a negative net export ratio in agriculture. That is,  $N_{imt} > 0$  and  $N_{iat} < 0$ .<sup>8</sup> Note that the trade patterns characterized in the following proposition is independent of the elasticity of substitution across sectors.

**Proposition 3 (Trade Patterns)** (i) Under free trade, the share of the total sector- $q$  expenditure on country  $i$ 's goods is the same across countries:  $\pi_{ijqt} = \pi_{jjqt}$ . (iv) If country  $i$  has a comparative advantage in manufacturing, the fraction of its total manufacturing expenditure that is on imports is smaller than the fraction of its agriculture expenditure on imports, i.e.,  $\pi_{ijm} < \pi_{ija}$  and  $\pi_{iim} > \pi_{iia}$ . (iii) The net export ratio is positive for the sector of comparative advantage in each country.

### 4.1.3 Labor Allocations

We begin with the services labor share. The market clearing condition for the non-tradable service good requires that  $C_{ist} = A_{ist}L_{ist} = X_{ist}w_{it}L_{it}/P_{ist}$ . Thus, we have

$$l_{ist} = \frac{L_{ist}}{L_{it}} = \frac{w_{it}L_{ist}}{w_{it}L_{it}} = \frac{P_{ist}C_{ist}}{w_{it}L_{it}} = X_{ist}. \quad (23)$$

In the open economy, the non-tradable sector's labor share equals its expenditure share — just as in the closed economy. However, the services labor share is higher in the open economy because the services expenditure share is higher in the open economy.

We then examine the tradable sector labor shares. Country 1's manufacturing income equals manufacturing expenditures of both countries on its manufacturing goods:  $w_{1t}L_{1mt} = \pi_{11mt}P_{1mt}C_{1mt} + \pi_{21mt}P_{2mt}C_{2mt}$ . This implies

$$l_{1mt} = \frac{L_{1mt}}{L_{1t}} = \pi_{11mt}X_{1mt} + \pi_{21mt}X_{2mt} \frac{w_{2t}L_{2t}}{w_{1t}L_{1t}}. \quad (24)$$

Country 1's manufacturing labor share depends on the manufacturing expenditure shares of both countries. In addition, it depends on the share of country 1's manufacturing spending that is on domestic goods, and on the share of country 2's manufacturing spending that is imported. Also, it depends on the relative size of the two economies. If country 1 is considerably smaller than country 2, then the impact of the open economy on the manufacturing labor share will be larger in country 1 than in country 2.

Substituting  $1 - \pi_{12mt}$  for  $\pi_{11mt}$ , we can rewrite (24) as follows:

$$l_{1mt} = X_{1mt} + \frac{\pi_{21mt}X_{2mt}w_{2t}L_{2t} - \pi_{12mt}X_{1mt}w_{1t}L_{1t}}{w_{1t}L_{1t}} = X_{1mt} + N_{1mt}. \quad (25)$$

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<sup>8</sup>Proofs are delegated to the appendix.

That is, country 1's manufacturing labor share equals its manufacturing expenditure share plus its manufacturing net export ratio. Thus, the tight link between preferences and production in the closed economy no longer holds in the open economy. A similar expression can be derived for the agriculture labor share.

In sum, the labor share of sector  $q = \{a, m\}$  in country  $i$  and period  $t$  is given by

$$l_{igt} = X_{igt} + N_{igt}. \quad (26)$$

The second term  $N_{igt}$  captures the direct contribution of international trade to structural change. There is an indirect contribution of the open economy on structural change through its effect on the prices and the expenditure shares  $X_{igt}$ . If country  $i$  has a comparative advantage in manufacturing, its agriculture labor share is lower in the open economy, both because its agriculture expenditure share is lower and because its agriculture net export ratio is negative in the open economy. Its manufacturing labor share cannot be signed, because, even though the manufacturing net export ratio is positive, the manufacturing expenditure share in the open economy relative to the closed economy cannot be signed. We summarize our findings in the following proposition.

**Proposition 4 (Labor Shares)** *Assume  $\epsilon < 0$ . (i) The services labor share is higher in the open economy than in the closed economy in both countries. (ii) The labor share of tradable sector  $q$  in country  $i$  is the sum of country  $i$ 's expenditure share on sector  $q$  goods and country  $i$ 's net export ratio in sector  $q$ . (iii) If country  $i$  has comparative disadvantage in sector  $q$ , the labor share of sector  $q$  is lower in the open economy than in the closed economy.*

It is often noted that trade sometimes acts like a productivity shock in a closed economy. By facilitating a reallocation of resources, openness to trade leads to an increase in overall output, even though overall inputs have not changed. For the effect of an open economy on the expenditure shares, this logic is useful, as the productivity shock interpretation for the tradable sectors helps us understand why agriculture's expenditure share falls and services' expenditure share rises (when the elasticity of substitution is less than one). This logic, however, does not offer a complete picture for thinking about structural change because in an open economy, sectoral employment is also determined by foreign demand for domestic goods. In addition, comparative advantage needs to be taken into account; one sector will experience an increase in employment owing to trade, while the other sector will experience a decrease, even though both experienced a productivity boost.

#### 4.1.4 Dynamics of Structural Change

We now study the dynamics of structural change in the open economy. Similarly as in the closed economy, the growth rate of the services labor share in country  $i$  is given by

$$\hat{l}_{ist} = \hat{X}_{ist}. \quad (27)$$

Different from the closed-economy implications, the growth rate of the labor share of tradable sector  $q$  in country  $i$  is given by

$$\hat{l}_{igt} = \frac{X_{igt}}{l_{igt}} \hat{X}_{igt} + \frac{N_{igt}}{l_{igt}} \hat{N}_{igt}. \quad (28)$$

We call the first term on the right-hand side the “expenditure” effect and the second term the “trade” effect, though trade also affects the expenditure shares indirectly through its impact on prices.

Suppose that the elasticity of substitution equals one. Then, the sectoral expenditure share is constant over time:  $\hat{X}_{igt} = 0$ . The labor share of tradable sector  $q$  is given by

$$\hat{l}_{igt} = \frac{N_{igt}}{\omega_m + N_{igt}} \hat{N}_{igt}. \quad (29)$$

As long as the trade effect is non-zero, we will have structural change over time, unlike in the closed economy case. In particular, when country  $i$  has a growing comparative advantage in manufacturing with  $\frac{A_{imt+1}}{A_{jmt+1}} > \frac{A_{imt}}{A_{jmt}} > \frac{A_{iat}}{A_{jat}} = \frac{A_{iat+1}}{A_{jat+1}}$ , we find that  $\hat{N}_{imt} > 0$ .<sup>9</sup> This implies that the trade effect is positive since  $N_{imt} > 0$ . Thus, country  $i$ ’s manufacturing labor share rises even when the elasticity of substitution is one.

Now let’s turn to the case in which the elasticity of substitution is less than one. As mentioned in the introduction, one of the central facts of structural change is the change in the sectoral allocation of employment over time. As a country develops, the agriculture share of employment falls, the services share rises, and the manufacturing share follows a hump pattern. One of the key contributions of NP is that it can provide an explanation for the hump even in the context of balanced aggregate growth. However, as we discussed earlier, it relies on a scenario in which manufacturing does not have the highest productivity growth, which is counterfactual for many emerging market countries that have undergone rapid structural change.

In an open economy, there is a scenario that can generate a hump even if manufacturing has the highest productivity growth. The story would go as follows. Initially, with fast

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<sup>9</sup>For details see the appendix.

growing comparative advantage in manufacturing, the trade effect is positive and increasing. Labor shifts towards the manufacturing sector to produce goods to satisfy increased global demand. This inflow of labor more than offsets the outflow owing to a declining expenditure share. The labor share of manufacturing increases. Over time, the trade effect, while remaining positive, diminishes as the economy moves closer to complete specialization. Once one country is completely specialized in the sector of comparative advantage, then, the expenditure effect dominates and the manufacturing labor share declines. This will be seen more clearly through the example presented in the next subsection.

## 4.2 Example

We illustrate the workings of the model with an example. In this example, we have a small country and a large country; country 1's labor force is one-tenth of country 2's. The initial sectoral productivity levels are the same in the per-capita term across countries.<sup>10</sup> Manufacturing TFP grows at 2 percent in country 1 but at 1 percent in country 2. Agriculture TFP grows at 1 percent in country 1 but at 2 percent in country 2. Thus, over time, country 1 has an increasing comparative advantage in manufacturing, and country 2 has an increasing comparative advantage in agriculture. In both countries, services TFP is constant over time. The elasticity of substitution is set to 0.5 and  $\omega_q$  is set at 1/3 for each sector. The parameters  $\sigma$ ,  $\eta$ , and  $\beta$  are irrelevant. Table 1 summarizes these parameters.

Table 1: Parameter Values

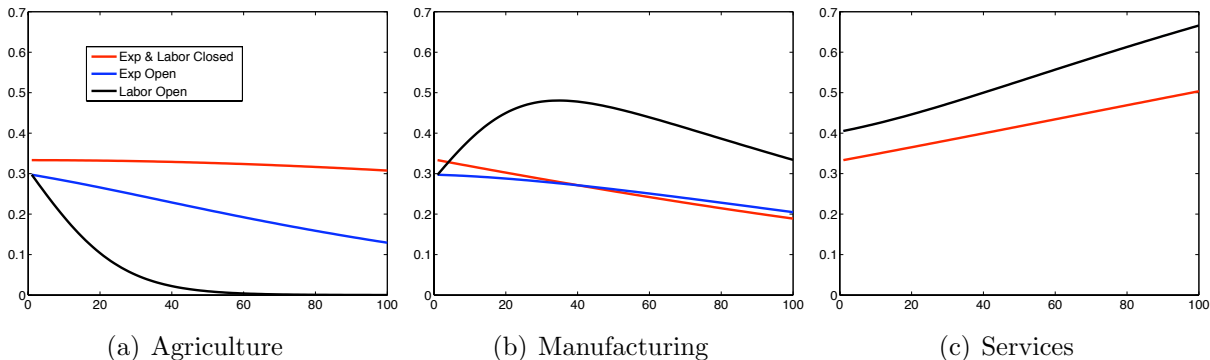
Preferences		
$\epsilon = -1.0$	$\omega_a = \omega_m = \omega_s = 1/3$	
Labor Endowment		
$L_{10} = 1$	$L_{20} = 10$	$\hat{L}_{1t} = \hat{L}_{2t} = 1.0$
Sectoral Productivities		
$\theta = 4.0$	$A_{1a0} = A_{1m0} = A_{1s0} = 1.0$	$A_{2a0} = A_{2m0} = A_{2s0} = (\frac{L_{20}}{L_{10}})^{1/\theta}$
$\hat{A}_{1at} = \hat{A}_{2mt} = 1.01$	$\hat{A}_{1mt} = \hat{A}_{2at} = 1.02$	$\hat{A}_{1st} = \hat{A}_{2st} = 1.0$

Figure 1 illustrates structural change in country 1 in both the open and closed economy cases. The closed economy is shown in red lines. In the closed economy, the employment shares of agriculture and manufacturing both decline, while that of services increases, monotonically. This is because the relative price of the composite agriculture and manufactured goods both decline, and with an elasticity of substitution less than one, this

<sup>10</sup>In a one-sector Eaton-Kortum model, the relative wage rate will be one if the two countries have the same per-capita productivity. In our multi-sector environment, the relative wage rate depends on the expenditure shares across sectors and across countries, in addition to the relative per-capita productivity. In this example, the initial relative wage rate turns out to be close to one, though not exactly one.

implies declining expenditure shares in these two sectors. Lastly, the labor shares equal the expenditure shares in the closed economy.

Figure 1: Structural Change in Country 1



In the open economy, however, the picture is quite different. The expenditure shares are shown in blue lines and the labor shares are shown in black lines. The agriculture expenditure share is lower and declines at a faster rate, and the services expenditure share is higher and rises at a faster rate in the open economy than in the closed economy.<sup>11</sup> The manufacturing expenditure share declines at a slower rate in the open economy, though its level in the open relative to closed economy is ambiguous. These patterns are driven by the fact that country 1 has a comparative advantage in manufacturing.

The agriculture labor share declines even faster than the expenditure share, again due to comparative disadvantage in agriculture. The manufacturing labor share, on the other hand, follows a hump pattern. The increasing comparative advantage in manufacturing over time means the trade effect is positive — an increasing fraction of labor is devoted to manufacturing for exports. Initially, the trade effect is stronger than the expenditure effect, and the manufacturing labor share increases. However, the strength of the trade effect diminishes over time, and is eventually dominated by the expenditure effect. This is the source of the peak and then subsequent decline in the manufacturing labor share. This eventual domination can be seen in panel (a) of Figure 1 and 3: country 2 ends up producing almost all the agriculture goods. Hence, labor in country 1 is allocated to only two sectors, services and manufacturing. As the services sector is growing, owing to its increasing relative price, the manufacturing sector must be shrinking.

Figure 2 presents the structural change patterns for country 2. Because the country's trading partner is small, the open economy patterns are similar to the closed economy patterns. However, the manufacturing sector shows a steeper decline, and the agriculture

<sup>11</sup>We observe only two lines in the services panel because the expenditure and labor shares coincide with each other.

sector shows a slower decline in the open economy than in the closed economy. Even relatively small economies can impact the pace of structural change of large economies.

Figure 2: Structural Change in Country 2

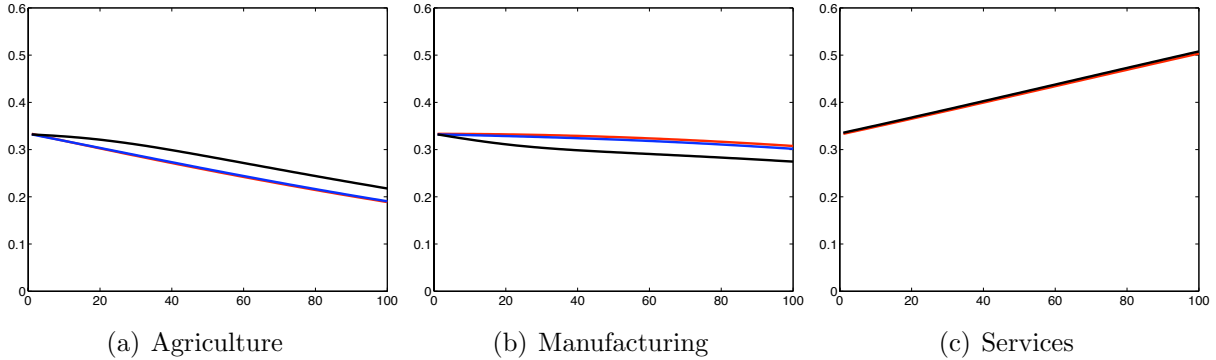
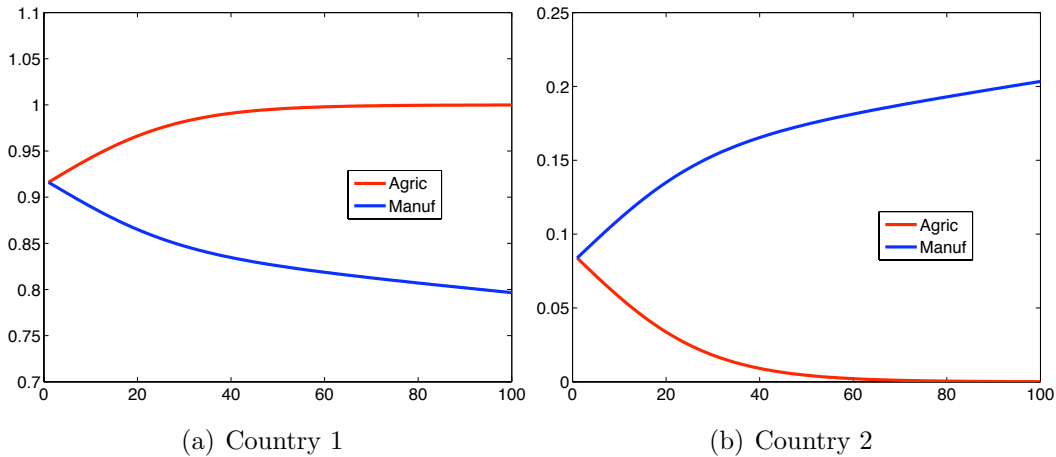


Figure 3 illustrates the trade patterns. The import shares of the smaller country 1 are high initially. Over time, owing to the increasing comparative advantage in manufacturing and increasing comparative disadvantage in agriculture, country 1 imports fewer manufactured goods and more agriculture goods. In the latter sector, as mentioned above, eventually, almost all consumption goods are imported. Country 2 imports an increasing share of its manufactured goods expenditure over time, but its expenditure shares on manufactured goods is declining over time. The net effect is the product of the expenditure share and the import share. At some point, total manufactured imports from country 1 diminish, which contributes to declining manufacturing labor in country 1.

Figure 3: Import Shares

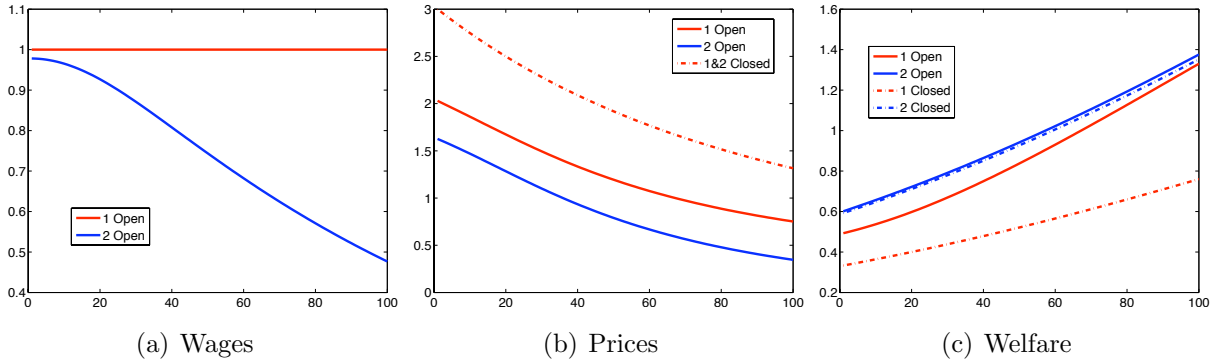


Finally, Figure 4 addresses welfare implications. Begin with wages shown in panel (a), where country 1's wage is the numeraire. Country 1's wage, relative to country 2's, rises

over time. To understand this, it is useful to first remember that, owing to symmetry in the parameters, if the two countries were of the same size, the relative wage would be constant. When both countries experience an increasing comparative advantage, they produce more of the same good (*intensive margin*) and more goods (*extensive margin*). The rise in the intensive margin tends to lower the wage more than the rise in the extensive margin. Since country 2 produces almost all the goods in period 1 due to its size, the intensive margin increases faster than the extensive margin as its comparative advantage rises. In contrast, the extensive margin rises faster than the intensive margin in country 1. Hence, the relative wage will rise. Panel (b) shows that both countries' price levels are lower than in the closed economy case, which is the effect of opening up to trade. As productivity in the tradable sectors grows over time, prices fall further.

Panel (c) illustrates the welfare effects over time. Welfare is measured as the wage rate divided by the overall price level. The two dashed lines illustrate the closed economy case. They grow at the same rate. This is a result of the symmetry between the two countries between agriculture and manufacturing. The two solid lines illustrate the open economy case. Note that opening up to trade provides a large boost to country 1, because it now has access to country 2's goods. By contrast, country 2 does not receive as much of a boost, owing to country 1's small size, and hence, a lower fraction of goods available to be imported. Over time, country 1 narrows the welfare 'gap' with country 2 by about 0.2 percent per year.

Figure 4: Wages, Prices, and Welfare



## 5 Extensions

We now relax, one at a time, three key assumptions in our model: homothetic preferences, free trade, and no intermediate goods. We show that our main results continue to hold.

### 5.1 Non-homothetic Preferences

The most common way that structural change has been modeled in the past is by using preferences that capture Engel's law, the fact that the food share of consumption diminishes as a country develops. In other words, the income elasticity of demand for food is less than one, and for at least one other sector, it is greater than one. The following non-homothetic preference specification encompasses Engel's law:

$$U(C_{iat}, C_{imt}, C_{ist}) = \omega_a \log(C_{iat} - L_{it}\bar{c}_a) + \omega_m \log(C_{imt} - L_{it}\bar{c}_m) + \omega_s \log(C_{ist} - L_{it}\bar{c}_s). \quad (30)$$

If  $\bar{c}_q > 0$ , we interpret  $\bar{c}_q$  as a per-capita subsistence requirement for sector  $q$  goods. This will generate an income elasticity of demand less than one. If, on the other hand,  $\bar{c}_q < 0$ , then the income elasticity of demand for the sector  $q$  good is larger than one.

We maintain the CES functional form for aggregating individual goods into the composite sectoral goods. This preserves the expressions for the sectoral prices.<sup>12</sup> For much of the analysis below, we will take  $\bar{c}_a > 0$ ,  $\bar{c}_m = 0$ , and  $\bar{c}_s < 0$ . This formulation is similar to that in Kongsamut, Rebelo, and Xie (2001). The sectoral consumption expenditure shares are given by

$$X_{iat} = \frac{P_{iat}C_{iat}}{w_{it}L_{it}} = \omega_a + \frac{(1 - \omega_a)\bar{c}_a P_{iat} - \omega_a \bar{c}_s P_{ist}}{w_{it}}, \quad (31)$$

$$X_{imt} = \frac{P_{imt}C_{imt}}{w_{it}L_{it}} = \omega_m + \frac{-\omega_m \bar{c}_s P_{ist} - \omega_m \bar{c}_a P_{iat}}{w_{it}}, \quad (32)$$

$$X_{ist} = \frac{P_{ist}C_{ist}}{w_{it}L_{it}} = \omega_s + \frac{(1 - \omega_s)\bar{c}_s P_{ist} - \omega_s \bar{c}_a P_{iat}}{w_{it}}. \quad (33)$$

We first examine the closed economy. As before, the labor shares equal the expenditure shares. Given the relationship between prices and productivities, we have

$$l_{ia} = \omega_a + \frac{(1 - \omega_a)\bar{c}_a}{A_{iat}} - \frac{\omega_a \bar{c}_s}{A_{ist}}, \quad (34)$$

$$l_{im} = \omega_m - \omega_m \left( \frac{\bar{c}_s}{A_{ist}} + \frac{\bar{c}_a}{A_{iat}} \right), \quad (35)$$

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<sup>12</sup>However, the price index for the overall consumption good will be different from (13).

$$l_{is} = \omega_s + \frac{(1 - \omega_s)\bar{c}_s}{A_{ist}} - \frac{\omega_s\bar{c}_a}{A_{iat}}. \quad (36)$$

Thus, because  $\bar{c}_a > 0$  and  $\bar{c}_s < 0$ , the agriculture labor share is greater than  $\omega_a$ , but decreases as productivities rise and countries get richer. The services labor share is always lower than  $\omega_s$ , and increases as productivities rise and countries get richer. The manufacturing labor share depends on the labor requirements to produce the minimum consumption requirement of services and agriculture goods. When countries become sufficiently rich, all the labor shares (and expenditure shares) converge to the appropriate  $\omega_q$ .

In the special case in which  $\bar{c}_a = -\bar{c}_s$  and all sectors have the same productivity  $\{A_{it}\}$ , we have

$$l_{ia} = \omega_a + \frac{\bar{c}_a}{A_{it}}, \quad (37)$$

$$l_{im} = \omega_m, \quad (38)$$

$$l_{is} = \omega_s + \frac{\bar{c}_s}{A_{it}}. \quad (39)$$

The above equations show that in the closed economy, there is structural change, even when all sectors have the same productivity and the elasticity of substitution between sectors is one.

Now consider the open economy. Prices have the same expressions as before. In particular, the agriculture and manufacturing composite sectoral goods prices are given by (20). As before, the tradable composite sectoral prices relative to the wage rate are lower in the open economy than in the closed economy (in each country), and the services price relative to the wage rate is unchanged.

Expenditure shares have the same expressions as in the closed economy following equation (31)-(33). Given the prices in the open economy, it is straightforward to see that the agriculture expenditure share is lower, while the manufacturing and the services expenditure shares are higher, compared to the closed economy. We can interpret the expenditure shares in an open economy as what would happen in a closed economy that experienced a positive TFP shock to the agriculture and manufacturing sectors.

The labor share of each sector depends on the trade pattern, in addition to the expenditure share, as in our benchmark model. We continue to assume that country 1 has a comparative advantage in manufacturing, that is  $A_{1m}/A_{2m} > A_{1a}/A_{2a}$ . The expression for the manufacturing labor share in country 1 is the same as 26. The expressions for the agriculture and services labor shares are also the same as before. Thus, our first key result, Proposition 3(ii), is preserved in the presence of non-homothetic preferences. It is also straightforward to show that  $N_{1mt} > 0$  and  $N_{1at} < 0$ , as before.

We now turn to the dynamics of the labor shares. Can we replicate Proposition 4 (iii)?

We again focus on a case in which each country's comparative advantage increases over time. We begin with the expression for the growth rate of the manufacturing labor share, (28). Relative to the benchmark model, we make one additional assumption. We assume that the productivities in agriculture and services in period  $t + 1$  are the same as those in period  $t$  in each country. We continue to assume that  $\frac{A_{1mt+1}}{A_{2mt+1}} > \frac{A_{1mt}}{A_{2mt}}$ . From here, following the same approach as in section 4.2, we can show that  $\hat{N}_{1mt} > 0$ . Hence, it is possible to generate a hump pattern, even in the presence of non-homothetic preferences. The essential intuition is that Engel's law provides a force for an increasing services labor share, and a possibly increasing manufacturing labor share, as incomes rise. The low elasticity of substitution provides a force for the manufacturing labor share to decline over time as prices and expenditure shares fall. The trade effect reinforces the Engel's law effect in manufacturing. Overall, it is, in some sense, easier to get the rising part of the hump, but a little more difficult to get the declining part of the hump, when non-homothetic preferences of this type are introduced.

## 5.2 Trade Costs

We now consider an intermediate case between frictionless trade and autarky, a case with positive, but finite, trade costs. When agriculture or manufacturing goods are shipped abroad, they incur trade costs, which include tariff rates, transportation costs, and other barriers to trade. We model all of these costs as iceberg costs. Specifically, if one unit of manufacturing good  $z$  is shipped from country  $j$ , then  $1/\tau_{ijm}$  units arrive in country  $i$ . Similarly,  $\tau_{ija}$  is the gross trade cost incurred from shipping one unit of the agriculture good from country  $j$  to country  $i$ . We assume that trade costs within a country are zero, i.e.,  $\tau_{11a} = \tau_{22a} = 1$  and  $\tau_{11m} = \tau_{22m} = 1$ .

The period  $t$  price of the composite sectoral good  $q = \{a, m\}$  in country  $i$  is given by

$$P_{iqt} = \left[ (w_{it}/A_{iqt})^{-\theta} + (\tau_{ijqt}w_{jt}/A_{jqt})^{-\theta} \right]^{-\frac{1}{\theta}}, \quad (40)$$

We modify our definition of comparative advantage to take into account trade costs. Specifically, country  $i$  has a comparative advantage in manufacturing if and only if

$$\tau_{ijm}A_{im}/A_{jm} > \tau_{ija}A_{ia}/A_{ja}. \quad (41)$$

We can define comparative advantage in agriculture for country  $i$  similarly. Clearly, if country  $i$  has a comparative advantage in manufacturing, it has a comparative disadvantage in agriculture, and vice versa. However, with trade costs, if country 1 has a comparative

advantage in manufacturing, it is not necessarily true that country 2 has comparative advantage in agriculture. This is only true either without trade costs, or when the trade costs are identical across the two sectors.<sup>13</sup> It is straightforward to show that Proposition 1 holds as before with “open economy” interpreted as an economy with positive but finite trade costs.

As trade costs only influence prices, then expenditure shares  $X_{iqt}$ , have the same functional form as before. The share of country  $i$ 's expenditure on sector  $q$  goods from country  $j$ ,  $\pi_{ijq}$ , is given by

$$\pi_{ijq} = \frac{(\tau_{ijq}w_j/A_{jq})^{-\theta}}{(\tau_{ijq}w_j/A_{jq})^{-\theta} + (w_i/A_{iq})^{-\theta}}. \quad (42)$$

Again, it is straightforward to demonstrate that Proposition 2 holds under trade costs under the same interpretation of open economy as above.

Exports and imports have the same general formulation as before, but now that they are defined inclusive of trade costs. Manufacturing exports of country  $i$  to country  $j$  are given by  $EX_{imt} = \pi_{jim}X_{jm}w_jL_j$ . Manufacturing imports of country  $i$  from country  $j$  are given by  $IM_{im} = \pi_{ijm}X_{im}w_iL_i$ . The balanced trade condition implies  $NX_{im} + NX_{ia} = 0$ .

We complete the characterization of the model with trade costs by deriving the sectoral labor allocations. These are unchanged once it is recognized that expenditures are now inclusive of trade costs. Hence, we still have the same expressions, (23), (24), and (26), as before.

We now turn to an alternative way of generating structural change: from lower trade barriers. Lowering trade barriers might lead to structural change in the open economy even without any change in productivities. Consider a simple example where the two countries have identical and constant sectoral productivities and identical and constant labor supplies. Productivities are the same across sectors, and  $\omega_a = \omega_m = \omega_s$ . Assume that the trade barriers are symmetric across countries, i.e.,  $\tau_{12at} = \tau_{21at} = \tau_{at}$  and  $\tau_{12mt} = \tau_{21mt} = \tau_{mt}$ . Also, assume that the trade barriers are the same across the two sectors in period 1:  $\tau_{a1} = \tau_{m1}$ . The trade barrier in the agriculture sector is constant over time, while the trade barrier in the manufacture sector declines over time. Specifically,  $\tau_{mt} - 1 = (\tau_{m1} - 1)\gamma_\tau^t$  for any  $t$  with  $\gamma_\tau < 1$ .

In the closed economy, the sectoral prices are constant over time, given the constant productivities. Also, the expenditure shares are equal across sectors and constant over time. The same is true for the labor shares. In the open economy, given the symmetry across the two countries, the wage rates are the same in both countries. Moreover, the sectoral prices are equal across countries given the identical productivities. Relative to the closed

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<sup>13</sup>For more discussion on this topic, see Deardorff (2004).

economy, the agriculture price and the manufacturing price decrease and the service price remains unchanged. Given the time path of the trade barriers, the manufacturing prices in both countries decline over time to the free-trade price, while the agriculture price remains constant over time. Consequently, both the agriculture sector and the service sector have an increasing expenditure share over time, while the manufacturing sector has a declining expenditure share in both countries. Labor shares equal expenditure shares in this case, despite the open economy, because there is balanced trade in each sector.

### 5.3 Intermediate goods

To introduce intermediate goods in a tractable way, we assume that each sector's output is produced from labor and intermediates, and the sector's output is used for consumption and as an intermediate in the same sector's production. In other words, there are no cross-sectoral intermediate input uses. Thus, country  $i$ 's production function for services is given by:

$$Y_{ist} = \psi A_{ist} L_{ist}^\alpha M_{ist}^{1-\alpha}, \quad (43)$$

where  $\psi = \alpha^{-\alpha} (1 - \alpha)^{\alpha-1}$ . Output  $Y_{ist}$  is used for consumption or as an intermediate to produce services. Hence, the services market equilibrium condition is:

$$Y_{ist} = C_{ist} + M_{ist}, \quad (44)$$

In each tradable sector, there is a composite intermediate good that has the same functional form as the composite final good:

$$M_{igt} = \left( \int_0^1 m_{igt}(z)^\eta dz \right)^{1/\eta}, \quad (45)$$

Country  $i$ 's production function for good  $z$  in sector  $q \in [a, m]$  is:

$$y_{igt}(z) = \psi A_{igt}(z) l_{igt}(z)^\alpha M_{igt}(z)^{1-\alpha}, \quad (46)$$

where  $M_{igt}(z)$  is the use of the composite intermediate good  $M_{igt}$  to make good  $z$ . The goods market equilibrium conditions are given by:

$$y_{1mt}(z) + y_{2mt}(z) = c_{1mt}(z) + c_{2mt}(z) + m_{1mt}(z) + m_{2mt}(z), \quad \forall z \in [0, 1]. \quad (47)$$

$$y_{1at}(z) + y_{2at}(z) = c_{1at}(z) + c_{2at}(z) + m_{1at}(z) + m_{2at}(z), \quad \forall z \in [0, 1]. \quad (48)$$

$$M_{imt} = \int_0^1 M_{imt}(z) dz, \quad (49)$$

$$M_{iat} = \int_0^1 M_{iat}(z)dz, \quad (50)$$

The prices of the sectoral goods in country  $i$  are given by:  $P_{ist} = w_{it}/A_{ist}^{\frac{1}{\alpha}}$  and

$$P_{iqt} = \left[ (w_{it}^\alpha/A_{iqt})^{-\theta} + (w_{jt}^\alpha/A_{jqt})^{-\theta} \right]^{-\frac{1}{\alpha\theta}}, \quad (51)$$

The share of country  $i$ 's expenditure on sector  $q$  goods from country  $j$ ,  $\pi_{ijq}$ , is given by

$$\pi_{ijq} = \frac{(w_j^\alpha/A_{jq})^{-\theta}}{(w_j^\alpha/A_{jq})^{-\theta} + (w_i^\alpha/A_{iq})^{-\theta}}. \quad (52)$$

Country  $i$ 's exports of manufactured goods in period  $t$  is given by  $EX_{imt} = \frac{1}{\alpha}(\pi_{jimt}X_{jmt}w_{jt}L_{jt})$ .

We now turn to the labor allocations. It is easy to show that the labor share in services is the same as before:  $l_{ist} = L_{ist}/L_{it} = X_{ist}$ . The equilibrium condition equating income to value-added from expenditure for country 1's manufacturing sector is given by:  $w_{1t}L_{1mt} = \alpha(\pi_{11mt}P_{1mt}(C_{1mt} + M_{1mt}) + \pi_{21mt}P_{2mt}(C_{2mt} + M_{2mt}))$ . Simplifying yields the same expression for the labor share as before:

$$l_{1mt} = L_{1mt}/L_{1t} = \pi_{11mt}X_{1mt} + \pi_{21mt}X_{2mt}w_{2t}L_{2t}/(w_{1t}L_{1t}). \quad (53)$$

Hence, all of the results from before go through in the presence of intermediate goods.

## 6 Conclusion

To be added

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## Appendix

If country 1 has a comparative advantage in manufacturing, then country 1 has a positive manufacturing net export ratio but a negative agriculture net export ratio.

We first solve for the relative wage rate  $\bar{w}_{qt}$  between country 1 and 2 that would equate sector- $q$  exports and imports in either country. Setting  $\pi_{12qt} = 1 - \pi_{21qt}$ , we derive

$$\bar{w}_{qt} = \left( \frac{X_{2qt} L_{2t}}{X_{1qt} L_{1t}} \right)^{\frac{1}{1+\theta}} \left( \frac{A_{1qt}}{A_{2qt}} \right)^{\frac{\theta}{1+\theta}}. \quad (54)$$

A relative wage rate higher than  $\bar{w}_{qt}$  would imply a trade deficit in sector  $q$  of country 1, and vice versa.

If country 1 has a comparative advantage in manufacturing, then it must be the case that  $\bar{w}_{mt} > \bar{w}_{at}$  because

$$\frac{\bar{w}_{mt}}{\bar{w}_{at}} = \left( \frac{X_{2mt} X_{1at}}{X_{2at} X_{1mt}} \right)^{\frac{1}{1+\theta}} \left( \frac{A_{1mt} A_{2at}}{A_{2mt} A_{1at}} \right)^{\frac{\theta}{1+\theta}} = \left( \frac{A_{1mt} A_{2at}}{A_{2mt} A_{1at}} \right)^{\frac{\theta}{1+\theta}}. \quad (55)$$

In equilibrium, trade must be balanced. This implies that the equilibrium ratio of wage rates  $w_t = \frac{w_{1t}}{w_{2t}}$  must lie between  $[\bar{w}_{at}, \bar{w}_{mt}]$ . Under this equilibrium relative wage rate  $w_t$ , it must be the case that  $N_{1mt} > 0$  and  $N_{1at} < 0$ . *Q.E.D.*

Assume that the elasticity of substitution is one and  $\frac{A_{1mt+1}}{A_{2mt+1}} > \frac{A_{1mt}}{A_{2mt}} > \frac{A_{1at}}{A_{2at}} = \frac{A_{1at+1}}{A_{2at+1}}$ . Then country 1's manufacturing net export ratio rises in period  $t + 1$ , i.e.,  $N_{1mt+1} > N_{1mt} > 0$ .

Suppose that the relative wage rate in period  $t + 1$  is the same as the one in period  $t$ .<sup>14</sup> Then, in period  $t + 1$ , country 1's manufacturing net export ratio rises as the productivity ratio rises, i.e.,  $N_{1mt+1} > N_{1mt} > 0$ . On the other hand, country 1's agriculture net export ratio remains constant from period  $t$  to  $t + 1$  because the productivity ratio and the relative wage rate are constant across the two periods, i.e.,  $N_{1at+1} = N_{1at} < 0$ . Thus, our assumption of the constant relative wage rate implies a violation of the balanced trade condition in period  $t + 1$ :  $N_{1mt+1} + N_{1at+1} > 0$  if  $N_{1mt} + N_{1at} = 0$ . Thus, the equilibrium relative wage rate has to rise in period  $t + 1$  to balance trade. This implies that the agriculture net export ratio decreases and manufacturing net export ratio increases in period  $t + 1$ . Thus, we have  $N_{1mt+1} > N_{1mt} > 0$ . *Q.E.D.*

<sup>14</sup>The equilibrium wage ratio  $\frac{w_{1t+1}}{w_{2t+1}}$  does not have a closed form solution.