

# Exchange Rate Regime, Optimal Debt Composition and Hedging in Small Open Economies: Empirical Evidence from Brazil

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## Abstract

This paper develops a model of the choice between local and foreign currency debt by firms facing exchange rate risk and hedging possibilities in small open economies. The model shows that the currency composition of debt and the optimal level of hedging are both endogenously determined as optimal firms' responses to a tradeoff between the lower cost of borrowing in foreign debt and the higher risk involved due to exchange rate uncertainty. Both debt composition and hedging depend on common factors such as foreign exchange risk and financial default, interest rates, the size of collateral and costs of exchange rate risk management. Results of the model are broadly consistent with lending and hedging behavior of the corporate sector in small open economies recently hit by a currency crisis. In particular, the model is able to explain why, unlike predictions of previous work in the literature of currency crisis, the collapse of the fixed exchange rate regime in Brazil -in early 1999- did not cause a major change in the currency composition of debt and the hedging behavior of the corporate sector.

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## I. Introduction

Recent currency crises were mainly characterized by the presence of currency mismatches between assets and liabilities and inadequate hedging in the balance sheets of the corporate sector.<sup>1</sup> This mismatch between foreign currency liabilities and domestic currency denominated assets in firm balance sheets has been argued to be the root cause of the large output collapses following currency depreciations.<sup>2</sup> Under a fixed exchange rate regime, firms understood fixed exchange rates to be a guarantee and failed to insure their foreign exposure. A direct implication of this line of reasoning is that once the exchange rate is allowed to float, firms will recognize their exposure, and foreign currency loans will be viewed as more costly so that firms reduce their foreign currency borrowing. Firms that still opted to borrow internationally would have incentives to hedge their foreign exposure. Unlike these predictions, firm level evidence from Brazil over the period of 1996-2001, suggests that the collapse of the fixed exchange rate regime in Brazil, in early 1999, did not cause a major change in the currency composition of debt and the hedging behavior of the corporate sector. This paper attempts to offer an explanation to this apparently surprising result.

To study this phenomenon the paper develops a theoretical framework that examines the firm's choice of local and foreign currency debt in the presence of exchange rate risk and hedging possibilities. The model is able to explain why some firms with access to foreign capital markets borrow in foreign currency and hedge their exchange risk exposure while others do not. Currency composition of debt and hedging operations are joint decisions and depend on common factors such as exchange rate risk and financial default, interest rates, the size of collateral and costs of foreign currency risk management. When the economy moves from fixed to float exchange rates firms change financing policies and the population of firms exposed to foreign exchange risk is altered. Firms with insufficient collateral and those unable to afford buying hedge lose access to

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<sup>1</sup> A currency mismatch occurs when a large fraction of firms' debt is denominated in foreign currency while income and assets are denominated in domestic currency. Empirical support of currency mismatches and exchange rate exposure is found by Burnside et al (2001) in the Asian crisis, Tesar and Dominguez (2001) in 8 non-industrialized and emerging markets and Bonomo et al (2003).

<sup>2</sup> To see more on the balance sheet explanation of currency crises see Krugman (1999) and Aghion, Bacchetta and Banerjee (2001).

capital markets. Firms with high enough collateral and those able to hedge increase their foreign debt. Firms with fairly enough collateral but unable to hedge borrow less in foreign currency; turn to domestic banks and are monitored in order to maintain their access to foreign capital markets. With a macroeconomic environment characterized by a moderate probability of currency depreciation and somewhat costless hedging, these changes in the population of firms can offset each other so that the currency composition of debt and hedging activities do not vary significantly across regimes.

The model predictions are broadly consistent with lending and hedging behavior of the corporate sector in small open economies recently hit by a currency crisis. The key element driving the model results is the type of incentives the firm faces in the presence of exchange rate risk. Foreign currency debt is preferred to domestic debt provided by local banks because banks lend funds at a higher interest rate that compensate for fixed monitoring costs. Although less expensive, foreign debt is more risky due to exchange rate uncertainty. In choosing their currency composition of debt the firm faces this trade off between costs and risks and finds that depending on the likelihood of currency devaluation and the costs involved in setting up currency forwards different financing policies are available. When affordable, hedging is another form of collateral expanding firms' debt capacity. Furthermore, firms heavily indebted in foreign currency are not necessarily exposed to exchange rate risk if they have enough collateral or are able to hedge the currency risk through forward markets.

The paper builds on the model developed by Holmstrom and Tirole (1997) adapted to the context of a small open economy. The paper with the analytical framework most closely related to this paper is Martinez and Werner (2001). They also extend the Holmstrom and Tirole model to the small open economy case and find that before the Mexican crisis in 1994, the decision of borrowing in pesos or dollars depended on the exchange rate regime due to an implicit guarantee given by the government under fixed exchange rates. However, these authors treat the exchange rate as a deterministic variable so that no hedging strategies on the firm side are discussed. Arguably, their model captures only part of the story; there is no discussion on how the interaction between domestic and dollar debt changes and how the population of firms is altered when the economy moves to a float regime and firms are able to hedge. In their paper, as in most of

the previous literature, it is implicit that large amounts of foreign currency debt represent a high degree of exposure of firms to exchange rate risk.

The model results are also consistent with previous empirical findings by Rossi (2004) and Bonomo et al (2003) using microeconomic data from Brazil. Bonomo et al (2003) find evidence of currency mismatches during the fixed exchange rate period. Rossi (2004) follows a market portfolio approach to estimate the determinants of firms' exchange rate exposure. Using financial data from 165 Brazilian companies over the period 1996 to 2002, Rossi finds that the adoption of the floating regime reduced the foreign vulnerability of the corporate sector by having a negative impact on firms' foreign borrowing and a positive impact on hedging.

The remainder of the paper is organized as follows. Section II presents the main empirical facts and the motivation for the analysis. Section III describes the model of optimal debt allocation and hedging at the firm level. Section IV offers the main results of the model under flexible and fixed exchange rates. Section V concludes.

## **II. Empirical evidence on foreign currency exposure and hedging: Brazil 1996-2001**

In this section firm-level data on Brazilian firms is examined before and after the currency crisis in 1999. As other Latin American countries, Brazil suffered from unexpected reversals in capital flows after subsequent crises in Mexico (1994), East Asia (1997) and Russia (1998). The macroeconomic adjustment forced by substantial capital outflows at the end of 1998 brought large and persistent swings in the exchange rate that finally led to the collapse of the crawling peg and a sharp devaluation of the real of 69 percent in January 1999.

Firm-level data correspond to financial information for 350 companies publicly traded in the Sao Paulo Stock Exchange Market over the period of 1996-2001.<sup>3</sup> This

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<sup>3</sup> This period is chosen because it provides valuable information to a comparative analysis on firm's behavior regarding financing and risk management policies before and after the currency crisis in 1999, and also because disclosure of information on derivatives was mandatory only after 1995.

information was collected directly from annual financial reports. Data contain information on balance sheet variables such as the assets, the currency composition of debt and shareholder's equity. Data on hedging transactions consists on year-end notional value of currency derivatives (forwards, futures, swaps and options) obtained from the explanatory notes to financial statements. Financial and state-owned firms are excluded because of their different motivation for using currency derivatives.

Table I provides information on the debt composition and hedging activities of firms across exchange rate regimes. The currency composition of debt measured by the ratio of dollar-denominated debt to total debt slightly decreases (from 43 to 38 percent) after the implementation of the float regime but on average it remains stable around 40 percent across regimes. Hedging operations, approximated by the fraction of currency derivatives to dollar debt, increased during the float regime from 3 to 4 percent. Interestingly, hedging operations were already observed during the fixed exchange rate period for some companies.

**Table I Brazil: Currency Composition of Debt and Hedging Operations: (in means) 1/**

	Fixed ER			Floating ER			Average	
	1996	1997	1998	1999	2000	2001	Fixed	Floating
Dollar Debt/Total Debt	39.1	44.0	46.0	40.2	39.6	37.3	43.2	38.2
Dollar Derivatives/Total Debt	0.4	2.5	3.0	2.3	2.6	4.2	2.6	4.1

1/ Corrected for valuation effect (dollar variables valued at exchange rate of year-end 1996). Source: Brazil Securities and Exchange Commission (CVM). Financial annual reports and explanatory notes to financial statements of companies publicly traded at the Sao Paulo Stock Exchange Market

Table II presents some interesting features regarding the hedging behavior of the corporate sector in Brazil. Most financial hedging transactions correspond to currency swap contracts. Unlike evidence for U.S. large non-financial corporations, which report currency forwards and options as the most commonly used tool to manage exchange rate exposure, firms in Brazil seem to prefer currency swaps, effectively converting foreign debt into domestic debt by simultaneous transactions in the spot and the forward markets.

This is consistent with the view that unlike U.S. companies, firms in small open economies of developing countries are mainly exposed to exchange rate risk coming from foreign debt rather than from foreign sales. Moreover, broad preference for currency swaps is also consistent with costly hedging, since a swap reduces transaction costs by allowing companies to arrange in only one contract what may take several transactions (e.g. forward contracts) to replicate.<sup>4</sup>

**Table II: Brazil: Hedging operations (in means)  
(Millions of US\$)**

	Fixed ER			Floating ER			Average	
	1996	1997	1998	1999	2000	2001	Fixed	Floating
Total Currency Derivatives	1.7	7.0	9.2	11.5	14.6	23.0	6.0	16.4
Currency forwards and futures	0.0	0.5	0.1	0.5	0.9	0.8	0.2	0.7
Currency swaps	1.7	6.3	9.1	11.0	11.3	20.9	5.7	14.4
Currency options	0.0	0.2	0.0	0.0	2.3	1.3	0.1	1.2

Source: Brazil Securities and Exchange Commission (CVM). Explanatory notes to financial statements of companies publicly traded at the Sao Paulo Stock Exchange Market

Regarding the firms' foreign exposure over time, Table III shows the number of firms holding financial debt in domestic and foreign currency and those using currency derivatives to hedge their foreign exposure. These data also provides evidence of minor changes in lending and hedging behavior. The number of firms borrowing in both currencies and the number of firms borrowing only in domestic currency both slightly increased. The number of firms that hedge using currency derivatives notoriously increases after the collapse of the exchange rate regime but, interestingly, more than half of the number of firms in the sample remains unhedged during the float regime.

<sup>4</sup> Bonomo et al (2003) pointed out that Brazilian firms prefer currency swaps because they can obtain advantageous swap contracts from local banks. Banks, in turn, are able to offer these contracts because they are not exposed to exchange rate risk since they hold government bonds indexed to the dollar in their portfolios. According to these authors, in the end hedge appears to be provided by government through bank's intermediation.

**Table III. Brazil: Exchange rate risk exposure overtime**

	1996	Fixed ER		Floating ER		
		1997	1998	1999	2000	2001
<u>Total Number of firms</u>	275	296	348	350	328	306
1. Number of firms with no financial debt	12	9	19	18	19	20
Percent of total firms	4%	3%	5%	5%	6%	7%
2. Number of firms with only domestic debt	33	34	39	37	44	45
Percent of total firms	12%	11%	11%	11%	13%	15%
3. Number of firms with dollar and domestic debt	109	139	181	200	188	196
Percent of total firms	40%	47%	52%	57%	57%	64%
Firms hedged using dollar derivatives	2	10	26	27	38	58
Firms not hedged	107	129	155	173	150	138
4. Number of firms with only dollar debt	3	3	16	9	19	17
Firms hedged using dollar derivatives	0	0	0	0	1	0
Firms not hedged	3	3	16	9	18	17
5. Number of firms with unknown debt position	118	111	93	86	58	28
Percent of total firms	43%	38%	27%	25%	18%	9%

Source: Brazil Securities Exchange Commission (CVM). Financial annual reports and explanatory notes to financial statements of companies publicly traded at the Sao Paulo Stock Exchange Market

Overall, the evidence presented in this section seems to support the view that there are no significant changes in the lending and hedging patterns of the Brazilian corporate sector during the period 1996-2001, that is, before and after the currency crisis. As will be shown in the next section, predictions of the model of optimal debt allocation and hedging are consistent with this empirical evidence.

### III. A Model of Optimal Currency Composition of Debt and Hedging

Consider a small open economy described by a two-date model. The economy is populated by a continuum of risk-neutral firms, domestic banks and foreign banks. Firms are run by wealth-constrained entrepreneurs who need to raise funds to cover their investment outlays. Firms' investment projects can be financed by borrowing in either domestic currency from local banks or in foreign currency from foreign banks. Given the uncertainty about the exchange rate, firms can choose to hedge their foreign exchange risk by signing forward contracts offered by local banks. At date  $t=0$  firms sign debt contracts and make investment, borrowing and hedging decisions. At date  $t=1$ , exchange rate and investment returns are realized and claims are settled. Agents are protected by limited liability so that no party can end up with negative payoffs.

#### *Firms' Investment Projects*

Each firm has access to a project requiring an investment of size  $I > 0$  at date  $t=0$ , which yields a verifiable return in domestic currency  $R$  in case of success and nothing in case of failure. Firms differ only in their initial capital  $A$  (which is publicly observable). The distribution of firms is described by the cumulative distribution function  $F(A)$ . It will be assumed that  $A < I$ , so that firms need funding to undertake their investments.

There are three types of investment projects as described in Figure 1: a good project with a high probability of success  $P_G$ , and a bad and a worse projects each with the same probability of success  $P_B$  ( $P_G > P_B$ ). The bad project gives a low private benefit  $b$  and the worse project gives a high benefit  $B$ , with  $B > b > 0$ . Firms face a moral hazard problem in choosing a project. In the absence of proper incentives or outside monitoring, entrepreneurs can divert resources by deliberately reducing the probability of success of a project (from  $P_G$  to  $P_B$ ) to enjoy the private benefit.

Figure 1: Three types of Investment Projects

Project	Good	Bad	Worse
Probability of success	$P_H$	$P_B$	$P_B$
Private Benefit	0	$B$	$B$

Local banks can monitor firms to reduce the moral hazard problem and eliminate the worse project (B-project). Local banks face a fixed cost of monitoring  $C > 0$ . Foreign banks are uninformed investors (i.e. they are unable to monitor firms) and have access to alternative projects with a gross rate of return in domestic currency  $r^f$ . It is assumed that only the good project has a positive expected net present value (NPV), even if the private benefit of the firm is included:

$$P_G R - r^f I > 0 > P_B R - r^f I + B$$

### *Firms' Financing Decisions and Exchange Rate Risk*

Firms are represented by the index  $f$ ; domestic banks are represented by the index  $m$  and foreign banks are represented by the index  $u$ . At date  $t=0$  a firm invests all its funds  $A^5$  and sign debt contracts to borrow in domestic currency an amount  $I_m$  from local banks or in foreign currency an amount  $I_u$  from foreign investors so that:<sup>6</sup>

$$A + I_m + s_L I_u = I \quad (1)$$

where  $s_L > 0$  is the exchange rate at date  $t=0$ , quoted as  $s_L$  units of domestic currency per unit of foreign currency.

Notice that investment return and exchange rate are both uncertain. Investment projects are subject to two types of bad events: failure (economic default) or bankruptcy due to currency-led default (financial default). Exchange rate fluctuations can turn solvent firms into bankrupt firms even when they undertake successful projects. It will be assumed that in case of any type of default firms' creditors nothing.<sup>7</sup>

For simplicity, consider only two states of nature about the exchange rate: low (L) and high (H). At time  $t=0$ , the exchange rate is  $s_L$ . If the economy operates under a fixed exchange rate regime then at date  $t=1$ :  $s_1 = s_L$ . Under a floating exchange rate regime, at date  $t=1$  the exchange rate  $s_1$  can either depreciate to  $s_H > s_L$  with probability  $q$ , or remain

<sup>5</sup> It is assumed that internal funds have a higher expected rate of return than outside funds.

<sup>6</sup> There is no other source of funding (e.g. equity finance) in the model. As is the case of emerging economies in Asia and Latin America, a large part of the funds for investments are provided in the form of bank loans.

<sup>7</sup> In case of financial default the firm is declared bankrupt and its residual value goes to pay for bankruptcy costs.

unchanged with probability  $1-q$ . To avoid losses due to exchange rate fluctuations, companies may decide to hedge their foreign exchange risk by buying currency forward contracts.

Let  $F$  be the one-period forward exchange rate defined as units of domestic currency per unit of foreign currency. Given the possibility of economic and financial default, it will be assumed that forward markets are efficient so that  $F$  is the expected exchange rate at date  $t=0$  and is given by:<sup>8</sup>

$$F = \begin{cases} (1-q)s_L + qs_H & \text{if financially solvent in both states L,H} \\ s_L & \text{if financially solvent only in state L} \\ s_H & \text{if financially solvent only in state H} \end{cases} \quad (2)$$

Expression (2) states that forward sellers adjust the forward rate according to the risk of financial default so that over hedging (i.e. infinite hedging in case of zero transaction costs) is ruled out because the firm would be unable to buy any amount of hedging if it is solvent in only one state of the exchange rate. It will also be assumed that the hedging contract is offered by domestic banks so that payments on the hedging contracts, in either direction, are also contingent on the project succeeding and are netted out on average.<sup>9</sup> Therefore, in case of failure a firm will not pay out on the hedging contract if state  $s_L$  occurs but will not any payment either if state  $s_H$  occurs.

Let  $h$  be the amount of forward contracts, in dollars, purchased by a firm and  $\varphi$  the transaction costs per unit of forward contract. For simplicity, let  $\varphi$  be also expressed in domestic currency per unit of foreign currency. At date  $t=1$ ; profits from hedging operations in domestic currency are given by:

$$\Pi^H = h (s_1 - F) - \varphi |h|$$

Note that  $s_L \leq F \leq s_H$ . Using equation (ii), expected profits from hedging operations are therefore equal to:

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<sup>8</sup> As shown later, the forward rate is related to interest rates according to a covered interest parity condition.

<sup>9</sup> For simplicity, it is assumed that firms buy forward contracts and borrow from different domestic banks so that transaction costs of hedging do not go as revenues to domestic lenders.

$$E \Pi^H = -\phi |h|$$

In this economy, a debt contract must specify the amount of each participant's investment and the payments to each of them under all circumstances so that (i) if the project fails, it pays zero to all parties (ii) At date  $t=1$  project returns  $R$ , in case of success, plus profits from hedging operations in case of no financial default (if any) are divided among parties, from which the firm receives  $R_f(s_1) \geq 0$ , domestic banks receive  $R_m(s_1) \geq 0$ , foreign banks receive  $s_1 R_u(s_1) \geq 0$  and forward sellers receive  $\phi |h| \geq 0$ .

Firms that undertake a successful investment are unable to meet their financial obligations and become bankrupt at date  $t=1$  if:

$$R - R_f(s_1) - R_m(s_1) - s_1 R_u(s_1) + h(s_1 - F) - \phi |h| < 0, \text{ where } s_1 = s_L, s_H$$

Firms may decide to hedge their dollar debt in order to reduce the possibility of financial default and be solvent in both states of the exchange rate. Therefore, if solvent, the firm pays back its creditor their promised payments regardless of the exchange rate state, which means that  $R_m$  and  $R_u$  can both be written as independent of the exchange rate realization. Moreover, it will be assumed that the firm is the residual claimant in the debt contract so that whatever is left after paying outside obligations depends on the exchange rate realization. Then, firm payments will be written as  $R_f(s_L)$  and  $R_f(s_H)$  respectively.

Since financial default may occur when  $s_1 = s_L$  or  $s_1 = s_H$ , let  $1_L$  and  $1_H$  be indicator variables such that:

$$1_H = \begin{cases} 1 & \text{if solvent in state H} \\ 0 & \text{if default in state H} \end{cases}$$

$$1_L = \begin{cases} 1 & \text{if solvent in state L} \\ 0 & \text{if default in state L} \end{cases}$$

Accordingly, expected cash flows if the firm invests in the good project are:

$$\begin{array}{ll}
P_G [ R - R_m - s_H R_u ] & \text{if solvent in state H} \\
0 & \text{if default in state H} \\
P_G [ R - R_m - s_H R_u ] & \text{if solvent in state L} \\
0 & \text{if default in state L}
\end{array}$$

Expected cash flows if the firm undertakes the bad or the worse projects are similar except for the additional private benefit  $b$  or  $B$  and the probability of success  $P_B$  multiplying the expression in brackets.

### *Firm's Problem*

At date  $t=0$  a firm with initial capital  $A$  and investment  $I$ , chooses the currency composition of its debt ( $I_m$  and  $I_u$ ), creditors' payments ( $R_m$  and  $R_u$ ), its hedging amount ( $h$ ) and its expected payments  $R_f(s_L)$  and  $R_f(s_H)$  to maximize expected total profits:

$$E[\Pi_{TOT}] = P_G \{ (1-q)1_L [RI - R_m - s_L R_u] + q1_L [RI - R_m - s_H R_u] - \phi | h | \}$$

subject to resource constraints, incentive and participation constraints and non-negativity constraints (e.g. limited liability).

In setting up the problem in this way, it is assumed that the firm undertakes only good projects with probability of success  $P_G$ . This is the case because only good projects have a  $NPV > 0$  which means that if bad projects are undertaken no borrowing is obtained from creditors. Moreover, as will be seen below, firms are given enough incentives to be diligent and undertake good projects.

Depending on the exchange rate realization, when the firm is financially solvent, project returns  $R$  and profits from hedging operations are distributed among all the parties according to:

$$R_f(s_L) + R_m + s_L R_u = R + h(s_L - F) - \phi | h | \quad (3)$$

$$R_f(s_H) + R_m + s_H R_u = R + h(s_H - F) - \phi | h | \quad (4)$$

When the firm defaults in either state H or L it pays nothing to parties and the residual value of the firm goes to pay for bankruptcy costs.

Assume momentarily that the firm chooses whether to borrow or not in domestic currency from a local bank. Later, it will be shown that in equilibrium the firm must find out whether it has to borrow from a local bank or not, depending on the size of its initial assets and its hedging strategy. However, for the purpose of describing the firm's incentive constraint, this assumption is harmless.

Let  $1_m$  be an indicator variable such that  $1_m = 1$  if the firm borrows from the local bank (with monitoring) and  $1_m = 0$  if the firm obtains its funding directly from foreign banks and does not borrow from the local bank (without monitoring). Given the two states of nature of the exchange rate, the firm invests in a good project whenever:

$$P_G[(1-q)R_f(s_L) + qR_f(s_H)] \geq P_B[(1-q)R_f(s_L) + qR_f(s_H)] + 1_m b + (1-1_m)B$$

This is the firm's incentive constraint and can also be written as:

$$(1-q)R_f(s_L) + qR_f(s_H) \geq 1_m \frac{b}{\Delta p} + (1-1_m) \frac{B}{\Delta p} \quad (5)$$

where  $\Delta p = P_G - P_B > 0$ .

### *Domestic Bank Lending*

A local bank monitors the firm and finances the project in domestic currency if it receives an expected payment sufficient to pay for the fixed monitoring cost  $C$ :

$$P_G[1_L(1-q) + 1_H q]R_m - 1_m C \geq P_B[1_L(1-q) + 1_H q]R_m$$

Therefore, the bank's incentive constraint is given by

$$[1_L(1-q) + 1_H q]R_m \geq 1_m \frac{C}{\Delta p} \quad (6)$$

On the other hand, the bank is willing to finance a project if it receives at least a net expected return (after paying for monitoring costs) equal to the opportunity cost of its funds. The participation constraint for the bank is then given by:

$$P_G[1_L(1-q) + 1_H q]R_m - 1_m C \geq r^f I_m \quad (7)$$

Let  $r$  denote the domestic lending rate that a bank charges on  $I_m$  funds lent to the firm, defined as:

$$r = \frac{P_G [1_L (1 - q) + 1_H q] R_m}{I_m} \quad (8)$$

The minimum domestic rate of return  $r$  acceptable for a bank that decides to finance an investment project in domestic currency is determined by the condition,

$$\frac{P_G C}{\Delta p} - C = r^f \frac{P_G C}{r \Delta p} \quad (8)$$

This expression is obtained by combining (vi) through (viii), all holding with equality, and assuming that the bank lends to the project so that  $1_m = 1$ . This condition, in turn, implies

$$r = r^f \frac{P_G}{P_B}$$

This condition states that the cost of domestic funds incorporates a risk premium relative to the risk-free rate. At the minimum rate of return acceptable for a bank this risk premium is equal to the ratio of success probabilities of the good and bad projects.

Banks can borrow and lend internationally and are able to replicate a forward contract. Non-arbitrage conditions ensure that banks are indifferent between lending in domestic currency at  $r^f$ , converting these funds into foreign currency at time zero at the spot rate  $s_L$  and lending abroad in foreign currency where they earn  $r^*$ , and converting these funds back to domestic currency at the forward rate  $F$ . These transactions imply that

$$r^f = r^* \frac{F}{s_L} \quad (10)$$

This is a covered interest rate parity condition and can be also expressed in terms of the bank lending rate  $r$  and the international rate  $r^*$  as:

$$r = \frac{r^* P_G}{P_B} \frac{F}{s_L} \quad (11)$$

Notice that in this expression  $P_G > P_B$  and  $F > s_L$  so that the bank lending rate  $r$  is always greater than the international interest rate  $r^*$  and therefore foreign debt is always preferred to domestic debt. As a result, when they have to, firms want to borrow the least they can from a local bank and obtain the rest from foreign lenders.

Given the possibility of financial default the firm will be able to borrow in foreign currency if foreign banks are promised an expected payment greater than or equal to what they could obtain by investing their funds in international capital markets at the rate  $r^*$ . The foreign bank participation constraint in foreign currency is then:

$$P_G [1_L (1 - q) + 1_H q] R_u \geq r^* I_u \quad (12)$$

### *Firm's Profit Maximization*

Given the firm's initial assets  $A$ , its fixed investment  $I$ , the foreign rate of return  $r^*$ , the probability of currency depreciation  $q$  and the cost of hedging  $\phi$ , the firm's problem is to find variables  $h$ ,  $I_m$ ,  $I_u$ ,  $R_m$ ,  $R_u$ ,  $R_f(s_L)$ ,  $R_f(s_H)$ ,  $1_m$ ,  $1_L$  and  $1_H$  to:

Maximize  $E[\Pi_{TOT}] = P_G \{ (1 - q) 1_L [R - R_m - s_L R_u] + q 1_H [R - R_m - s_H R_u] - \phi | h | \}$   
subject to

$$A + I_m + s_L I_u = I \quad (1')$$

If firm is solvent:

$$R_f(s_L) + R_m + s_L R_u = R + h (s_L - F) - \phi | h | \quad (2')$$

$$R_f(s_H) + R_m + s_H R_u = R + h (s_H - F) - \phi | h | \quad (3')$$

$$(1 - q) R_f(s_L) + q R_f(s_H) \geq 1_m \frac{b}{\Delta p} + (1 - 1_m) \frac{B}{\Delta p} \quad (4')$$

$$[(1 - q) 1_L + 1_H q] R_m \geq 1_m \frac{C}{\Delta p} \quad (5')$$

$$P_G [(1 - q) 1_L + 1_H q] R_m - 1_m C \geq r^f I_m \quad (6')$$

$$P_G [(1 - q) 1_L + 1_H q] R_u \geq r^* I_u \quad (7')$$

$$R_f(s_H) \geq 0 \quad (8')$$

$$R_f(s_L) \geq 0 \quad (9')$$

Conditions (1') through (3') are resource constraints. Conditions (4') and (5') are the incentive compatibility constraints for the firm and the domestic bank. Conditions (6') and (7') are the participation constraints for domestic and foreign banks. Conditions (8') and (9') are non-negativity constraints for the firm's payments. Note that only one of these two non-negativity constraints will be binding when the firm defaults in one state of the exchange rate but is solvent in the other.

### *Assumptions*

The following assumptions will be used to support and equilibrium:

1.  $P_H R - r^f I - C > 0$
2.  $\frac{s_H - s_L}{s_L} (1 - q) < 1$

Assumption 1 ensures the economic viability of the good project by stating that it has a positive net present value per unit of investment after accounting for monitoring costs. Assumption 2 implies that currency depreciation ( $s_H/s_L - 1$ ) must occur with sufficiently high probability so that firms will have incentives to hedge their exchange rate risk.

### *Determination of Equilibrium*

There are four possible cases in the profit maximization problem of a representative firm. The maximized objective function is different depending on:

- i.  $1_H=1$  and  $1_L=1$  if the firm is solvent in both states
- ii.  $1_H=1$  and  $1_L=0$  if the firm is solvent only in state H
- iii.  $1_H=0$  and  $1_L=1$  if the firm is solvent only in state L
- iv.  $1_H=0$  and  $1_L=0$  if the firm is insolvent in both states

Case (iv) is the uninteresting case and is ruled out because it involves zero profits and no borrowing at all. Expected profits for the remaining three cases are respectively:

- i.  $E[\Pi_{TOT}] = P_G [R - R_m - FR_u - \phi | h |]$
- ii.  $E[\Pi_{TOT}] = P_G [q(R - R_m - s_H R_u) - \phi | h |]$
- iii.  $E[\Pi_{TOT}] = P_G [(1 - q)(R - R_m - s_L R_u) - \phi | h |]$

Case (ii) is also ruled out because it is always dominated by case (i). To see this, note that for any  $0 < q < 1$  then  $s_H > F$  and profits are greater in case (i) than in case (ii). Therefore, the relevant cases to evaluate are only (i) and (iii), which means that in equilibrium, the firm is always solvent in state L ( $1_L = 1$ ) and case (iii) of insolvency in state H happens only when the firm is not hedged at all<sup>10</sup>.

The previous analysis implies that in equilibrium there are 9 constraints to solve for 9 endogenous variables. Moreover, given that the firm is the residual claimant in debt contracts, it is straight forward to see that constraints (5') through (7') will be binding. This is the case because in order to maximize profits the firm chooses the minimum payments that make its creditors willing to participate financing the project.

A representative firm solves its maximization problem at  $t=0$  using the following algorithm:

- i. Suppose that given the lower cost of foreign debt the firm decides to borrow directly from foreign banks ( $1_m = 0$ ). Furthermore, suppose the firm judges it is financially solvent without hedging ( $1_H = 1$ ,  $1_L = 1$  and  $h = 0$ ). Conditions (5') and (6') jointly determine  $I_m = 0$  and  $R_m = 0$ . Given  $A$  and  $I$ , the firm finds  $I_u$  using (1) and then  $R_u$  is determined by (7'). With expected payments  $R_m$  and  $R_u$  already pinned down, constraint (3') is used to verify if the firm is solvent in state H.<sup>11</sup> If it is, then the firm can in fact borrow directly from foreign markets without hedging.
- ii. When the firm is not solvent in state H and decides not to hedge then foreign creditors adjust their expected payment.  $R_u$  depends on  $q$  and is bigger to compensate for the possibility of default. Constraint (8') is binding so that  $R_f(s_H) = 0$  (e.g. the residual

<sup>10</sup> Partly hedging is ruled out because in case of currency depreciation, this level of hedging is not sufficient to avoid default so that the firm is forced to default as if hedging were zero in the first place. With sufficiently small but positive costs of hedging the firm will prefer zero hedging to partly hedging.

<sup>11</sup> Two cases of solvency in state H are considered. First, the firm is solvent if it receives its expected payment in state H, that is  $R_f(s_H) = b/\Delta p$ , even after paying its creditors (constraints (8') and (9') are both non binding). A less stringent second criterion for solvency is when the firm at least guarantees repayment in state H even though it is left with nothing so that  $R_f(s_H) = 0$  (constraint (8') is binding but constraint (9') is non binding).

value of the firm goes to pay for bankruptcy costs if state H occurs). Constraint (2') gives the firm's own payment  $R_f(s_L)$ . If constraint (4') is met then borrowing in foreign currency without hedging is feasible. Otherwise, the firm could still borrow directly from foreign markets but must use forward contracts to reduce the possibility of default in state H.

- iii. If the firm hedges its foreign exchange risk via currency forwards then (2') and (3') combined with (8') and (9') determine an optimal range for hedging  $[\underline{h}, \bar{h}]$  given by:

$$\frac{s_H R_u + R_m - R}{s_H - F - \varphi} \leq h \leq \frac{R - R_m - s_L R_u}{F - s_L + \varphi}$$

Profit maximization implies that for any positive and small  $\varphi$  the firm chooses the minimum level to avoid default so that optimal hedging is  $h = \underline{h}$  with  $R_m = 0$ . As a result, constraint (8') is binding and  $R_f(s_H) = 0$ . Note that when  $\varphi = 0$  there are multiple solutions for the optimal hedge because the firm is indifferent choosing any value within the range  $[\underline{h}, \bar{h}]$ . If constraint (4') is met then borrowing in foreign currency and hedging is feasible. The firm determines whether hedging is optimal or not by comparing profits with the not hedging case. If constraint (4') is not met then the firm cannot borrow directly from foreign markets and must turn to domestic banks first.

- iv. Suppose the firm borrows from domestic banks ( $1_m = 1$ ) and believes it is financially solvent without hedging ( $1_H = 1, 1_L = 1$  and  $h = 0$ ). As before, constraints (5') and (6') jointly determine  $I_m$  and  $R_m$ . These two variables are now:

$$R_m = \frac{C}{\Delta p(1 - q + 1_H q)} \quad \text{and} \quad I_m = \frac{P_B s_L C}{r^* F \Delta p}$$

Notice that when the firm is solvent in state H,  $R_m$  does not depend on  $q$ . Given  $A$  and  $I$ , resource constraint (1) determines  $I_u$  and the currency composition of debt is pinned down. As in steps (ii) and (iii) of this algorithm, the firm can borrow now in both

currencies without hedging if it is solvent in state H or can face higher payments if it is not solvent in state H and does not hedge. Whether the firm must hedge or not depends on exogenous parameters (in particular  $q$  and  $\varphi$ ). The firm solves for its optimal hedging decision by comparing profits in each case. If the firm must hedge the optimal range for hedging is  $[\underline{h}, \bar{h}]$  with  $R_m > 0$ . As before, the minimum level  $h = \underline{h}$  is chosen when  $\varphi > 0$  or any level within  $[\underline{h}, \bar{h}]$  when  $\varphi = 0$ . Finally, if constraint (4') is not met even with a positive level of hedging then the firm is poorly capitalized (too small net worth) and will not be able to borrow at all.

The solution above shows that the currency composition of debt and the optimal level of hedging are both endogenously determined as optimal firms' responses to a tradeoff between the lower cost of borrowing in foreign debt and the higher risk involved due to exchange rate uncertainty. Foreign debt is preferred to domestic debt so that the firm always tries to borrow directly from foreign banks. The smaller the size of  $A$  is, the more the firm demands from international banks. Clearly, the firm cannot always borrow as much as it demands in foreign currency because exchange rate uncertainty makes foreign debt risky. Foreign banks demand collateral or hedging through currency forwards to ensure that the firm is solvent enough to repay its debt. As the algorithm shows, if the firm's initial net worth is not sufficient to meet the requirement then the firm must be monitored and borrow from domestic banks to have access to foreign markets. Depending on the size of its net worth  $A$ , the firm may also need to hedge to demonstrate financial solvency.

A key element in the distribution of incentives that the firm faces is the fact that hedging strategies are perfectly observed by creditors. An immediate implication of this feature of the model is that firms have incentives to hedge their exchange rate risk to reduce the probability of financial default. When affordable and useful, hedging increases profits by expanding the possibility of borrowing in foreign currency at a lower interest rate. Furthermore, hedging is a necessary but not a sufficient condition for financial solvency. For example, a firm with sufficiently high net worth can borrow only small amounts of foreign debt and be financially solvent in both L and H states without hedging.

Depending on exogenous parameters, there will be situations when hedging may turn to be useless or impossible to afford. In order to see this, let  $\bar{q}$  be probability of depreciation that makes  $\underline{h} = 0$  so that:

$$\bar{q} = 1 - \frac{\frac{b}{\Delta p} \frac{s_H}{(s_H - s_L)}}{R - \frac{C}{\Delta p}}$$

and  $\bar{\varphi}$  the transaction cost in forward markets above which, for any given level of positive  $q$ , hedging is irrelevant (e.g. forward contracts are too costly to provide insurance), then:

$$\bar{\varphi} = s_H - F = (1 - q)(s_H - s_L)$$

When the probability of depreciation is too high, say  $q > \bar{q}$ , then hedging is useless to protect the firm from the possibility of financial default driven by currency depreciation. On the other hand, when  $\varphi > \bar{\varphi}$  then hedging is not affordable.

#### *Minimum net worth requirements and hedging*

The previous analysis shows that the representative firm solves its profit maximization problem under two situations, when it hedges enough and when it does not hedge at all. In the latter case, two possible outcomes may arise: the firm is solvent even without hedging or the firm defaults if it is not hedged. Each of these situations exists for a company borrowing from domestic banks and for a company borrowing directly from foreign banks. These multiple cases imply that creditors demand different collateral levels (i.e. minimum net worth requirements) when the firm borrows from local banks and when it borrows directly from foreign markets. A detailed solution to solve for all these collateral requirements is presented in the appendix but for the purpose of analysis Table IV shows all of them.

**Table IV: Creditor's Minimum Net worth Requirements**

I. Firms borrowing in both domestic and foreign currency:

$$\underline{A}_H(r^*, q, \varphi) = I - \frac{P_B s_L}{r^*} \frac{C}{F \Delta p} - \frac{P_G}{r^*} \left[ R - \frac{C}{\Delta p} - \frac{b}{\Delta p(1-q)} \frac{(1-q)(s_H - s_L) - \varphi}{s_H - s_L} \right] \frac{s_L}{F + \varphi}$$

Optimally hedged

$$\underline{A}_{NH}(r^*, q) = I - \frac{P_B s_L}{r^*} \frac{C}{F \Delta p} - (1-q) \frac{P_G}{r^*} \left[ R - \frac{C}{\Delta p(1-q)} - \frac{b}{\Delta p(1-q)} \right]$$

Not hedged and not solvent in state H

$$\underline{A}_{SNH}(r^*, q) = I - \frac{P_B s_L}{r^*} \frac{C}{F \Delta p} - \frac{P_G s_L}{r^* s_H} \left[ R - \frac{C}{\Delta p} \right]$$

Not hedged but solvent in state H. Firm's own expected return is zero

$$\underline{A}_S(r^*, q) = I - \frac{P_B s_L}{r^*} \frac{C}{F \Delta p} - \frac{P_G s_L}{r^* s_H} \left[ R - \frac{b}{\Delta p} - \frac{C}{\Delta p} \right]$$

Not hedged but solvent in state H. Firm receives its expected return

$$A_B(r^*, q) = I - \frac{P_B s_L}{r^*} \frac{C}{F \Delta p}$$

Not hedged  
Firm borrows only in domestic currency

II. Firms borrowing only in foreign currency

$$\bar{A}_H(r^*, q, \varphi) = I - \frac{P_G}{r^*} \left[ R - \frac{B}{\Delta p(1-q)} \frac{(1-q)(s_H - s_L) - \varphi}{s_H - s_L} \right] \frac{s_L}{F + \varphi}$$

Optimally hedged

$$\bar{A}_{NH}(r^*, q) = I - (1-q) \frac{P_G}{r^*} \left[ R - \frac{B}{\Delta p(1-q)} \right]$$

Not hedged and not solvent in state H

$$\bar{A}_{SNH}(r^*) = I - \frac{P_G s_L}{r^* s_H} R$$

Not hedged but solvent in state H. Firm's own expected return is zero

$$\bar{A}_S(r^*) = I - \frac{P_G s_L}{r^* s_H} \left[ R - \frac{B}{\Delta p} \right]$$

Not hedged but solvent in state H. Firm obtains its expected return

In equilibrium, the firm compares its initial assets  $A$  with these cutoff levels to decide the optimal currency composition of its debt (e.g. how much to borrow from each source) and whether it should be hedged or not.

## IV. Model results

### *Equilibrium with costless hedging ( $\varphi = 0$ )*

Costless hedging represents a long run equilibrium in which currency forward markets are competitive and well-developed so that transaction costs are negligible. The next two results illustrate the optimal financing policies when the economy operates under a fixed exchange rate regime and under floating exchange rates as two separate steady state equilibriums.

***Lemma 1*** *In an economy with fixed exchange rates, that is when  $q=0$  and  $\varphi > 0$ , the optimal strategy for a firm is not to hedge its dollar debt.*

Proof: See Appendix

This basic result explains why a fixed exchange rate biases the currency composition of debt for some firm towards foreign currency debt and eliminates incentives to operate in forward markets. By fixing the exchange rate, government provides a form of public hedging or free risk management to the corporate sector by creating a perception of no foreign exchange risk.<sup>12</sup> Consequently, some firms with insufficient capital (that otherwise would have difficult access to international capital markets), or some others that should be monitored, are now able to obtain foreign currency debt without constraints. This situation creates a risky population of firms that would have incentives to borrow extensively in foreign currency without hedging. These firms are exposed to exchange rate risk and maintain currency mismatches in their balance sheets. As predicted by the balance sheet approach of currency crises, in the event of unexpected and large currency depreciation the corporate sector in this economy would face widespread bankruptcy.

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<sup>12</sup> When  $q=0$  and  $\varphi = 0$  the cutoff levels of the net worth ratios are at their lowest value and firms will be indifferent between hedging and not hedging. Although hedging becomes irrelevant, the level of optimal hedging is indeterminate and firms can choose any amount since there are no transaction costs.

**Lemma 2** *In an economy with floating exchange rates, when  $0 < q < \bar{q}$  and  $\varphi = 0$ , in equilibrium the optimal strategy for a firm with net worth  $A$  such that  $\underline{A}_H < A < \underline{A}_{SNH}$  or  $\bar{A}_H < A < \bar{A}_{SNH}$  is to hedge its dollar debt through currency forwards enough to avoid bankruptcy.*

Proof: See Appendix

According to this result, as long as the probability of currency depreciation is reasonably high so that firms can use forward contracts to deal with exchange rate risk, hedging makes it easier for the firm to obtain funding from foreign banks at a lower cost because it reduces the required collateral. Without transaction costs in forward markets hedging is always preferred to not hedging and any firm that is not solvent in state H has incentives to hedge enough to avoid default. This is the case when the firm uses a mixture of domestic and foreign debt or when it borrows only from foreign banks. Note also that this result states the benefits of hedging for firms that are not solvent in state H. Firms with net worth above the minimum requirement for solvency in state H do not need to hedge.

*Equilibrium with costly hedging  $\varphi > 0$*

In the previous analysis when  $\varphi = 0$  net worth requirements  $\underline{A}_{NH}$  and  $\bar{A}_{NH}$  are irrelevant because any firm is optimally hedged. This is not going to be the case when there are transaction costs in currency forward markets as stated in the next result.

**Proposition 1:** *With costly hedging and a positive probability of depreciation the optimal strategy for a representative firm regarding debt composition and hedging operation depend on  $q$  and  $\varphi$ , such that:*

- i. *When  $q \geq \bar{q}$  the firm borrows the least it can in domestic currency from local banks and borrow the rest in foreign currency from foreign banks and does not hedge its foreign debt.*

- ii. When  $0 < q < \bar{q}$  and  $\varphi < (1 - q)(s_H - s_L)$  the firm finds it optimal to borrow the least it can in domestic currency from local banks and the rest in foreign currency from foreign banks and hedge its foreign debt enough to avoid default.
- iii. When  $0 < q < \bar{q}$  and  $\varphi \geq (1 - q)(s_H - s_L)$  the firm finds it optimal to borrow the least it can in domestic currency from local banks and the rest in foreign currency from foreign banks and does not hedge their foreign debt.

Proof: See Appendix

To illustrate this result, notice that different combinations of  $q$  and  $\varphi$  imply different incentives for a firm deciding its optimal financing and hedging strategies. As the first case in proposition 2 states, extremely high probability of depreciation turns hedging into a worthless strategy to protect against exchange rate risk so that no firm participates in currency forward markets. A similar situation occurs in the third case when firms have incentives to hedge but forward contracts are too expensive or inexistent. Although these two situations look similar because in each case firm are not able to hedge, they are different and have different interpretations. Intuitively, higher values of both  $q$  and  $\varphi$  increase the collateral requirement making it more difficult to borrow in foreign currency. In the first situation,  $q$  is too high that the firm is forced to borrow only small amounts in foreign currency so that it can be solvent in state H without hedging. In the third case, however, the firm still borrows large enough amounts of foreign debt but is unable to hedge and therefore has a currency mismatch in its balance sheet. This firm would default in equilibrium if state H happens.<sup>13</sup>

The second case in proposition 1 is an intermediate situation and is relatively similar to that one described in lemma 2, so that by hedging optimally the firm is able to

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<sup>13</sup> In case i of proposition 2, regardless of  $\varphi$ ,  $q > \bar{q}$  makes  $\underline{A}_H > \underline{A}_{SNH}$  when the firm borrows in both domestic and foreign currency and  $\bar{A}_H > \bar{A}_{SNH}$  when the firm borrows only in foreign currency so the relevant minimum net worth required is  $\underline{A}_{SNH}$  and  $\bar{A}_{SNH}$  respectively. In case iii of proposition 2,  $\varphi > \bar{\varphi}$  also implies  $\underline{A}_H > \underline{A}_{SNH}$  and  $\bar{A}_H > \bar{A}_{SNH}$  but now solvency under state H matters so that depending on  $q$  the relevant minimum requirement can be  $\underline{A}_{SNH}$  or  $\underline{A}_{NH}$  when the firm borrows in both currencies and  $\bar{A}_{SNH}$  or  $\bar{A}_{NH}$  if the firm borrows only in foreign currency. However, in any of these cases, firms do not hedge.

finance its investment using foreign capital markets. A positive but moderate probability of depreciation and a positive small cost of hedging determine the following distribution of collateral requirements:

$$\underline{A}_H < \underline{A}_{NH} < \underline{A}_{SNH} < \underline{A}_S < \bar{A}_H < \bar{A}_{NH} < \bar{A}_{SNH} < \bar{A}_S$$

This ordering of net worth requirements define cutoff levels of an equilibrium segmentation of firms into different categories depending on their demand for bank loans and their hedging strategy, as shown in Figure 2. Well-capitalized firms with net worth  $A > \bar{A}_H$  finance their investment directly in foreign currency from international banks. Poorly capitalized firms with  $A < \underline{A}_H$  cannot invest at all since they have no access to any type of finance. In between, somewhat capitalized firms with  $\underline{A}_H < A < \bar{A}_H$  can invest only to the extent that they are monitored and domestic bank loans. Firms in this monitoring region finance their investment with a mixture of domestic and foreign debt. Whether firms hedge or not also depends on the size of their initial asset  $A$ . Well-capitalized firms need not hedge if  $A > \bar{A}_{SNH}$  but must hedge if  $\bar{A}_H < A < \bar{A}_{SNH}$ . Similarly, somewhat capitalized firms need not hedge if  $\underline{A}_{SNH} < A < \bar{A}_H$  but must hedge if  $\underline{A}_H < A < \underline{A}_{SNH}$ .

A typical firm within the monitoring region uses a mixture of domestic and foreign currency debt to finance its investment. However, firms with net worth  $A < \bar{A}_H$  but  $A + I_m >$  only demand bank loans in domestic currency and invest their excess funds outside the firm. The minimum net worth requirement for these firms is:

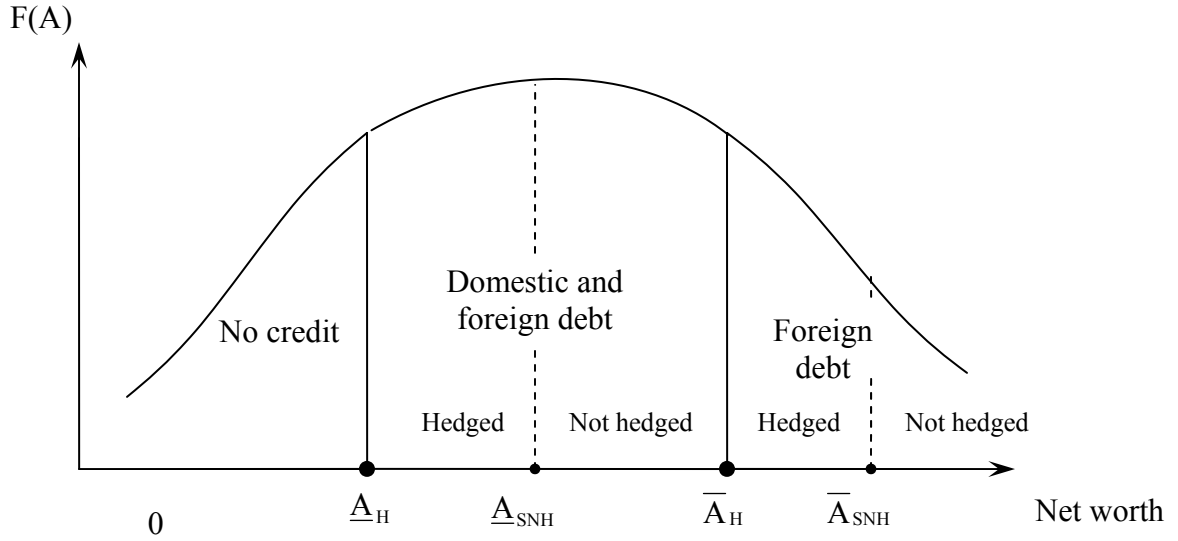
$$A_B(r^*, q) = I - \frac{P_B}{r^*} \frac{s_L}{F} \frac{C}{\Delta p} \quad (14)$$

Moreover  $\underline{A}_{SNH} < A_B < \bar{A}_H$ <sup>14</sup>, which means some firms within the monitoring region with net worth above  $A > A_B$  borrow in domestic currency only.

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<sup>14</sup> This is the result of a particular combination of parameters. It happens when monitoring cost  $C$  is sufficiently high so that local banks loan out relatively high amounts in domestic currency. In general, this does not have to be always the case. When domestic debt is small enough so that  $A_B > \bar{A}_H$  then  $A_B$  is no

**Figure 2: Equilibrium segmentation of firms**



Given this assumption, a more specific distribution of net worth requirements in the equilibrium segmentation can be:

$$\underline{A}_H < \underline{A}_{NH} < \underline{A}_{SNH} < \underline{A}_S < A_B < \bar{A}_H < \bar{A}_{NH} < \bar{A}_{SNH} < \bar{A}_S$$

Note that when costs of hedging are small enough  $\underline{A}_{NH}$  and  $\bar{A}_{NH}$  are irrelevant so they are not shown in Figure 2. Moreover, the relevant cutoff levels of net worth for firms that need not hedge are  $\underline{A}_{SNH}$  and  $\bar{A}_{SNH}$  so that both  $\underline{A}_S$  and  $\bar{A}_S$  are also omitted in Figure 2.

The equilibrium segmentation describes the distribution of firms in the economy according to their demand for foreign debt and their hedging decisions. This segmentation depends on exogenous parameters, in particular, those describing the macroeconomic environment such as interest rates, probability of currency depreciation and costs of hedging. When these parameters change, the segmentation is altered and the population of firms exposed to foreign exchange risk changes accordingly. The next result illustrates how different exchange rate regimes and different stages of development of forward markets determine a variety of allocations of domestic and foreign debt and also a distinctive hedging behavior.

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longer a relevant requirement. A firm with sufficiently high collateral, say with  $A_B > A > \bar{A}_H$  would never borrow only in domestic currency because it has the possibility to borrow only in foreign currency at a lower cost.

**Proposition 2:** *When the economy moves from fixed to floating exchange rates and in the float regime  $0 < q < \bar{q}$  and  $\varphi < (1 - q)(s_H - s_L)$ , all else equal:*

- i. Fewer firms obtain funding for their investment.*
- ii. More firms turn into local banks to finance their investment.*
- iii. Fewer firms finance their investment borrowing directly from foreign banks.*

Proof: See appendix

As an illustration of the changes regarding the currency composition of debt, notice that since  $I_m$  is a fixed amount, each firm in the economy demands the same minimum amount of domestic debt and the aggregate demand for domestic bank loans is then:

$$D_m(r^*, q) = [F[\bar{A}_H(r^*, q)] - F[\underline{A}_H(r^*, q)]] I_m(r^*, q)$$

where the individual demand  $I_m$  is written as a decreasing function of both  $r^*$  and  $q$ . On the other hand, the aggregate demand for foreign currency loans is given by:

$$D_u(r^*, q) = \int_{\underline{A}_H(r^*, q)}^{\bar{A}_H(r^*, q)} [I - I_m(r^*, q) - A] \partial F(A) + \int_{\bar{A}_H(r^*, q)}^{\infty} [I - A] \partial F(A)$$

Since  $r^*$  and  $q$  are exogenous parameters and assuming perfect competition in the domestic banking system, the supply of domestic funds is perfectly elastic at the domestic lending rate  $r$  and the supply of foreign loans is perfectly elastic at the international interest rate  $r^*$ , which means that  $D_m$  and  $D_u$  determine the aggregate amount of domestic and foreign lending in equilibrium.

An increase in  $q$  brought by the collapse of the fixed exchange rate regime has an ambiguous effect on  $D_m$  because both cutoff levels  $\underline{A}_H(r^*, q)$  and  $\bar{A}_H(r^*, q)$  increase and there are two opposing effects. A first group of firms with insufficient collateral and those unable to hedge cannot borrow are all dropped so that  $D_m$  decreases. Some other firms turn to domestic banks and increase  $D_m$  because they have insufficient collateral or are unable to hedge and cannot borrow only in foreign currency.

The impact of  $q$  on  $D_u$  is ambiguous as well. The group of firms that were previously borrowing in foreign debt and turned to domestic debt reduces their demand for foreign currency debt so that  $D_u$  drops. However, firms with fairly enough collateral and those able to hedge remain in the monitoring region and increase their demand for foreign debt because  $I_m$  decreases for them and, as a result,  $D_u$  increases. In sum, a number of firms borrow less in both currencies and some others borrow more in foreign currency. How these changes in the population of firms affect the aggregate currency composition of debt depends on the distribution function  $F(A)$ . The changes in the population of firms are illustrated in Figure 3.

**Figure 3: Changes in the Equilibrium Segmentation: from fixed to floating exchange rates**

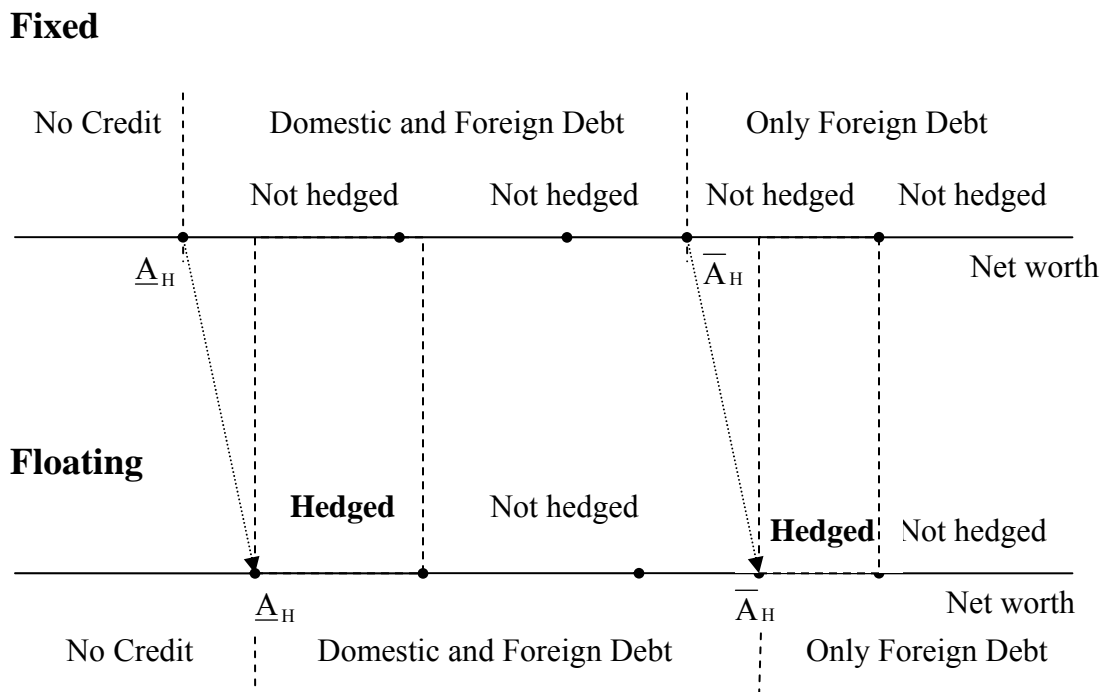


Figure 3 shows that the switch in the exchange rate regime shifts the monitoring region, making it more difficult for firms to borrow in foreign currency. The assumption behind these changes is that the probability of depreciation is relatively small so that a number of firms have incentives to switch from not being hedged during the fixed exchange rate regime to being hedged during the float regime. However, depending on

how large  $q$  and  $\varphi$  are, these changes can also be small. For example, as  $q$  increases and approximates to  $\bar{q}$  the group of firms that hedge using currency forwards, represented by the shadowed areas, becomes smaller and can be very small if  $\varphi$  is also sufficiently high as to prevent some firms from hedging.

### *The Brazilian Experience*

As a final illustration, consider how the results of the model match up with the currency crisis in Brazil in early 1999. In light of the model results, the lack of major changes in the lending and hedging behavior of the corporate sector in Brazil can be the result of a moderate increase in the probability of currency depreciation and the existence of somewhat costless hedging. After the float regime is adopted, some firms borrow less in both domestic and foreign currency and some others borrow more in foreign currency. Moreover, a group of firms borrowing in foreign currency need not hedge since they have enough collateral and are solvent even in the event of currency depreciation. These changes in the population of firms can offset each other so that the currency composition of debt and hedging activities do not vary significantly across regimes.

Needless to say, there are various other aspects of the macroeconomic environment as well as firm-specific characteristics conditioning the currency composition of lending and the hedging behavior of firms in reality. Moreover, in contrast to what the model assumes, a currency crisis most likely affects the firms' net worth so that the distribution of firms may not be constant across regimes. Furthermore, other parameters such as the probability of success and the investment payoff  $R$  are certainly different across firms and are also affected by the collapse of the exchange rate regime. How companies deal with higher foreign exchange risk definitely depend on changes in these variables, treated as invariant parameters in the model. For example, depending on specific characteristics such as export status, type of ownership or the existence of other sources of funding; firms can adopt hedging strategies other than the use of currency forwards. Nevertheless, the model developed in the paper suggests changes in the behavior of a representative firm and impacts on the population of firms that are broadly consistent with the empirical facts observed in recent currency crises in small open economies, and in particular, in Brazil during the period of 1996 to 2001.

## V. Conclusions

This paper introduces hedging decisions in a model of optimal debt allocation at the firm level to understand the sources of currency mismatch in the balance sheet of the corporate sector of countries that recently faced a currency crises. In particular, the model explains why some firms with access to foreign currency debt hedge their exchange risk exposure and others do not, as an optimal response to appropriate incentives given by the macroeconomic environment. Under fixed exchange rates firms borrow extensively in foreign currency and do not hedge because they have no incentives to do so given that government provides a type of free risk management. Under a float regime, when the probability of currency depreciation is moderate and hedging is affordable firms use currency forwards to hedge their exchange rare risk exposure and reduce the probability of financial default. Hedging works as collateral allowing hedged firms to expand their capacity to access foreign capital markets.

Despite the obvious limitations of a partial equilibrium analysis and some simplifying assumptions, the model is able to provide an analytical framework to determine endogenously the currency composition of credit and the optimal level of hedging at the firm level. Consistent with the empirical evidence in Brazil during 1996 to 2001, the model predicts that with a moderate probability of currency depreciation and somewhat costless hedging, the changes in the population of firms after the economy adopts a float regime can offset each other so that the currency composition of debt and hedging activities do not vary significantly across regimes.

While the currency composition of debt remained stable in Brazil after the collapse of the exchange rate regime, costless hedging operations seems to provide an effective vehicle to reduce foreign exposure without affecting significantly the aggregate levels of borrowing. A direct policy implication of the model is then the necessary emphasis that policymakers should give to the development of currency derivatives markets to help the corporate sector in smoothing the transition to a free-floating exchange rate regime.

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## Appendix

### Determination of firm's minimum net worth requirements in equilibrium

To find the minimum net worth requirement (collateral) under different hedging strategies suppose that instead of using its initial assets as an exogenous variable, the entrepreneur wants to determine the size of  $A$  necessary to maximize expected profits and meet all the constraints.  $A$  can then be treated as an additional endogenous variable that the entrepreneur finds in equilibrium and compare with her actual initial assets, renamed as  $A_0$ , to determine for example whether the firm is solvent without hedging, or whether the firm needs to borrow from local banks and be hedged, etc.

I. The firm borrows from local banks ( $l_m=1$ ). There are three different cases:

- a. Firm is solvent in both states only if hedged ( $l_H=1$  and  $l_L=1$ ). There are 9 constraints to solve for 8 unknowns ( $h, I_m, I_u, R_m, R_u, R_f(s_L), R_f(s_H), A$ ) so that one constraint is non-binding. Constraints (2') and (3') combined with (8') and (9') defines the range  $[\underline{h}, \bar{h}]$  for optimal hedging  $h$  and for any positive and small transaction cost  $\varphi$ , the firm chooses  $h = \underline{h}$ . This condition is equivalent to have constraint (8') binding and (9') non-binding. Therefore, the firm hedges enough to avoid default and it gets zero cash flow in the devaluation state. Having constraints (1') through (8') holding with equality, in equilibrium:

$$R_f(s_L) = \frac{b}{\Delta p(1-q)}, R_f(s_H) = 0, R_m = \frac{C}{\Delta p}, I_m = \frac{P_B s_L C}{r^* F \Delta p}$$

$$R_u = \frac{s_L}{F + \varphi} \left( R - \frac{b+C}{\Delta p} + \frac{b\varphi}{\Delta p(s_H - s_L)(1-q)} \right), I_u = \frac{P_G s_L}{r^* F} \left[ R - \frac{(b+c)}{\Delta p} \right]$$

$$h = \frac{1}{F + \varphi} \left[ R - \frac{C}{\Delta p} - \frac{b}{\Delta p} \frac{s_H}{(1-q)(s_H - s_L)} \right]$$

The minimum  $A$  required by foreign banks is:

$$\underline{A}_H(r^*, q, \varphi) = I - \frac{P_B s_L C}{r^* F \Delta p} - \frac{P_G}{r^*} \left[ R - \frac{C}{\Delta p} - \frac{b}{\Delta p(1-q)} \frac{(1-q)(s_H - s_L) - \varphi}{s_H - s_L} \right] \frac{s_L}{F + \varphi}$$

and total expected profits when  $A = \underline{A}_H$  are:

$$E[\Pi_{TOT}] = P_G \frac{b}{\Delta p} - \frac{P_G \varphi}{F + \varphi} \left[ R - \frac{C}{\Delta p} - \frac{b}{\Delta p} \frac{s_H}{(1-q)(s_H - s_L)} \right]$$

- b. Firm is solvent in both states even if it is not hedged: ( $1_H=1$ ,  $1_L=1$  and  $h=0$ ). This situation requires high enough collateral (i.e. higher than the minimum  $A$  when it must hedge to be solvent). The first criterion for solvency is that the firm is at least able to pay its debt even if it has to give up its own expected payment during the depreciation state, that is  $R_f(s_H)=0$ . A firm will be able to invest if in return it expects to get higher payments during the non-depreciation state. Therefore constraints (4') and (9') are not binding and there are 7 binding constraints to solve for 7 unknowns ( $I_m, I_u, R_m, R_u, R_f(s_L), R_f(s_H), A$ ). The equilibrium solutions in this case are:

$$R_f(s_L) = \left( R - \frac{C}{\Delta p} \right) \left( \frac{s_H - s_L}{s_H} \right), R_f(s_H) = 0, h=0, R_m = \frac{C}{\Delta p}, R_u = \frac{1}{s_H} \left( R - \frac{C}{\Delta p} \right)$$

$$I_m = \frac{P_B s_L}{r^* F} \frac{C}{\Delta p}, \quad I_u = \frac{1}{s_H} \frac{P_G}{r^*} \left[ R - \frac{C}{\Delta p} \right]$$

The minimum  $A$  required by foreign banks is now:

$$\underline{A}_{SNH}(r^*) = I - \frac{P_B s_L}{r^* F} \frac{C}{\Delta p} - \frac{P_G s_L}{r^* s_H} \left[ R - \frac{C}{\Delta p} \right]$$

As can be seen, when  $h>0$  then  $\underline{A}_H < \underline{A}_{SNH}$  which means that foreign banks would demand higher collateral compared to the case when firms must hedge to be solvent. Assumption 2 ensures that the firm gets a higher payment in the good state, that is,  $R_f(s_L) > b/[\Delta p(1-q)]$ , as expected. Total expected profits when  $A = \underline{A}_{SNH}$  are:

$$E[\Pi_{TOT}] = P_G (1-q) \left( R - \frac{C}{\Delta p} \right) \left( \frac{s_H - s_L}{s_H} \right)$$

- c. Firm is solvent in both states even if it is not hedged ( $1_H=1$ ,  $1_L=1$  and  $h=0$ ) but in this case the firm still gets its own expected payment during the depreciation state, that is,

$R_f(s_H) = b/\Delta p$  and the firm gets higher payments during the non-depreciation state. Unlike the previous case, now the collateral required is greater than  $\underline{A}_{SNH}$ . Constraints (4'), (8') and (9') are not binding so that there are 6 binding constraints to solve for 6 unknowns ( $I_m, I_u, R_m, R_u, R_f(s_L), A$ ). The equilibrium solutions in this case are:

$$R_f(s_L) = \left(R - \frac{C}{\Delta p}\right) \left(\frac{s_H - s_L}{s_H}\right) + \frac{1}{s_H} \frac{b}{\Delta p}, \quad R_f(s_H) = \frac{b}{\Delta p}, \quad h=0, \quad R_m = \frac{C}{\Delta p},$$

$$R_u = \frac{1}{s_H} \left(R - \frac{b}{\Delta p} - \frac{C}{\Delta p}\right), \quad I_m = \frac{P_B s_L}{r^* F} \frac{C}{\Delta p} \quad \text{and} \quad I_u = \frac{1}{s_H} \frac{P_G}{r^*} \left[R - \frac{b}{\Delta p} - \frac{C}{\Delta p}\right]$$

The minimum  $A$  required by foreign banks is now:

$$\underline{A}_S(r^*, q) = I - \frac{P_B s_L}{r^* F} \frac{C}{\Delta p} - \frac{P_G s_L}{r^* s_H} \left[R - \frac{b}{\Delta p} - \frac{C}{\Delta p}\right]$$

As can be seen,  $\underline{A}_{SNH} < \underline{A}_S$  which means that this is even higher collateral compared to the case when the firm solvent but gives up its own expected payment. As expected  $R_f(s_L) > b/[\Delta p(1-q)]$ . It will be assumed also that the monitoring technology is socially valuable<sup>15</sup>, then  $\underline{A}_H < \bar{A}_H$ ,  $\underline{A}_{NH} < \bar{A}_{NH}$  and  $\underline{A}_S < \bar{A}_S$ . Total expected profits when  $A = \underline{A}_S$  are:

$$E[\Pi_{TOT}] = P_G \left[ (1-q) \left(R - \frac{C}{\Delta p}\right) \left(\frac{s_H - s_L}{s_H}\right) + \frac{F}{s_H} \frac{b}{\Delta p} \right]$$

- d. Firm is not solvent if it is not hedged ( $1_H=0, 1_L=1$  and  $h=0$ ). Since the firm is not solvent in state H then (2') is ruled out and (8') is binding so that  $R_f(s_H)=0$ . Moreover, (9') is non-binding so that  $R_f(s_L) > 0$ . Therefore, there are 7 constraints to solve for 7 unknowns ( $I_m, I_u, R_m, R_u, R_f(s_L), R_f(s_H), A$ ). The equilibrium solution is:

$$R_f(s_L) = \frac{b}{\Delta p(1-q)}, \quad R_f(s_H) = 0, \quad h=0, \quad R_m = \frac{C}{\Delta p(1-q)}, \quad I_m = \frac{P_B s_L}{r^* F} \frac{C}{\Delta p}$$

<sup>15</sup> Monitoring is valuable when  $C[P_G - \frac{s_L}{F} P_B] < P_G[B-b]$

$$R_u = \frac{1}{s_L} \left( R - \frac{b+C}{\Delta p(1-q)} \right), I_u = (1-q) \frac{P_G}{r^*} \left[ R - \frac{(b+c)}{\Delta p} \right]$$

The minimum net worth requirement is:

$$\underline{A}_{NH}(r^*, q) = I - \frac{P_B}{r^*} \frac{s_L}{F} \frac{C}{\Delta p} - (1-q) \frac{P_G}{r^*} \left[ R - \frac{C}{\Delta p(1-q)} - \frac{b}{\Delta p(1-q)} \right]$$

and total expected profits when  $A = \underline{A}_{NH}$  are:  $E[\Pi_{TOT}] = P_G \frac{b}{\Delta p}$

II. Firm borrows directly from foreign banks ( $1_m=0$ ). There are also three different cases:

- a. Firm is solvent in both states if hedged ( $1_H=1$  and  $1_L=1$ ). There are 7 constraints to solve for 6 unknowns ( $h, I_u, R_u, R_f(s_L), R_f(s_H), A$ ) so that one constraint is non-binding. As before, constraints (2') and (3') combined with (8') and (9') define the range  $[\underline{h}, \bar{h}]$  for optimal hedging  $h$ . For any positive and small transaction cost  $\varphi$ , the firm chooses the minimum level to avoid default so that optimal hedging is  $h = \underline{h}$ . This result implies that constraint (8') is binding and (9') is non binding. The firm hedges enough to avoid default and it gets zero cash flow in the devaluation state. In equilibrium:

$$R_f(s_L) = \frac{B}{\Delta p(1-q)}, R_f(s_H) = 0, R_m = 0, I_m = 0, I_u = \frac{P_G}{r^*} \frac{s_L}{F} \left[ R - \frac{B}{\Delta p} \right]$$

$$R_u = \frac{s_L}{F + \varphi} \left( R - \frac{b+C}{\Delta p} + \frac{b\varphi}{\Delta p(s_H - s_L)(1-q)} \right), h = \frac{1}{F + \varphi} \left[ R - \frac{B}{\Delta p} \frac{s_H}{(1-q)(s_H - s_L)} \right]$$

The minimum  $A$  required by foreign banks is:

$$\bar{A}_H(r^*, q, \varphi) = I - \frac{P_G}{r^*} \left[ R - \frac{B}{\Delta p(1-q)} \frac{(1-q)(s_H - s_L) - \varphi}{s_H - s_L} \right] \frac{s_L}{F + \varphi}$$

and total expected profits when  $A = \bar{A}_H$  are:

$$E[\Pi_{TOT}] = P_G \frac{B}{\Delta p} - \frac{P_G \varphi}{F + \varphi} \left[ R - \frac{B}{\Delta p} \frac{s_H}{(1-q)(s_H - s_L)} \right]$$

- b. Firm is solvent in both states even if it is not hedged: ( $1_H=1$ ,  $1_L=1$  and  $h=0$ ). Intuitively again, this firm has high enough collateral (i.e. higher than the minimum  $A$  when it has to hedge to be solvent). As in the case of  $1_m=1$  now constraints (4') and (9') are not binding and there are 5 binding constraints to solve for 5 unknowns ( $I_u$ ,  $R_u$ ,  $R_f(s_L)$ ,  $R_f(s_H)$ ,  $A$ ). The equilibrium solutions are:

$$R_f(s_L) = R \left( \frac{s_H - s_L}{s_H} \right), R_f(s_H) = 0, h=0, R_m = 0, R_u = \frac{1}{s_H} R,$$

$$I_m = 0, I_u = \frac{1}{s_H} \frac{P_G}{r^*} R$$

The minimum  $A$  required by foreign banks is now:

$$\bar{A}_{SNH}(r^*) = I - \frac{P_G}{r^*} \frac{s_L}{s_H} R$$

and total expected profits when  $A = \bar{A}_{SNH}$  are:  $E[\Pi_{TOT}] = P_G \frac{B}{\Delta p}$

Note that when  $h>0$  then  $\bar{A}_H < \bar{A}_{SNH}$  which means that foreign banks demand higher collateral compared to the case when firms must hedge to be solvent. Assumption 2 also ensures that, in fact, the firm gets a higher payment in the good state, that is,  $R_f(s_L) > B/[\Delta p(1-q)]$ , as expected.

- c. Firm is solvent in both states even if it is not hedged as before ( $1_H=1$ ,  $1_L=1$  and  $h=0$ ) but in this case the firm still gets its own expected payment during the depreciation state, that is  $R_f(s_H) = b/\Delta p$  and the firm gets higher payments during the non-depreciation state. Unlike the previous case, now the collateral required is higher than  $\bar{A}_{SNH}$ . Constraints (4') and (8') and (9') are not binding and there are 4 binding constraints to solve for 4 unknowns ( $I_u$ ,  $R_u$ ,  $R_f(s_L)$ ,  $A$ ). The equilibrium solutions are:

$$R_f(s_L) = R \left( \frac{s_H - s_L}{s_H} \right) + \frac{1}{s_H} \frac{B}{\Delta p}, R_f(s_H) = \frac{B}{\Delta p}, h=0, R_m = 0, R_u = \frac{1}{s_H} \left( R - \frac{B}{\Delta p} \right)$$

$$I_m = 0, I_u = \frac{1}{s_H} \frac{P_G}{r^*} \left[ R - \frac{B}{\Delta p} \right]$$

The minimum A required by foreign banks is now:

$$\bar{A}_S(r^*) = I - \frac{P_G}{r^*} \frac{s_L}{s_H} \left[ R - \frac{B}{\Delta p} \right]$$

As can be seen,  $\bar{A}_{SNH} < \bar{A}_S$  and as expected  $R_f(s_L) > B/[\Delta p(1-q)]$ . Total expected profits when  $A = \underline{A}_S$  are:

$$E[\Pi_{TOT}] = P_G \left[ (1-q)R \left( \frac{s_H - s_L}{s_H} \right) + \frac{F}{s_H} \frac{B}{\Delta p} \right]$$

- d. Firm is not solvent if it is not hedged ( $1_H=0$ ,  $1_L=1$  and  $h=0$ ). Constraint (2') is ruled out and constraint (8') is binding so that  $R_f(s_H)=0$ . Constraint (9') is non-binding so that  $R_f(s_L) > 0$ . Therefore, there are 5 constraints to solve for 5 unknowns ( $I_u$ ,  $R_u$ ,  $R_f(s_L)$ ,  $R_f(s_H)$ ,  $A$ ). The equilibrium solution is:

$$R_f(s_L) = \frac{B}{\Delta p(1-q)}, \quad R_f(s_H) = 0, \quad h=0, \quad R_m = 0, \quad R_u = \frac{1}{s_L} \left( R - \frac{B}{\Delta p(1-q)} \right)$$

$$I_m = 0, \quad I_u = (1-q) \frac{P_G}{r^*} \left[ R - \frac{B}{\Delta p} \right]$$

and the minimum net worth requirement is:

$$\bar{A}_{NH}(r^*, q) = I - (1-q) \frac{P_G}{r^*} \left[ R - \frac{B}{\Delta p(1-q)} \right]$$

Total expected profits when  $A = \bar{A}_{NH}$  are:  $E[\Pi_{TOT}] = P_G \frac{B}{\Delta p}$

- III. Firm borrows only in domestic currency from local banks: then  $R_u = I_u = 0$ . It is straight forward to show that the firm is indifferent between hedging and not hedging if  $\varphi=0$ . Hence, without loss of generality, it can be concluded that firms are always solvent and do not hedge because they do not need to. Constraints (8') and (9') are non-binding and constraint (7') is not relevant. Profit maximizing firms pay domestic banks just the

least banks demand to participate so that constraints (5') and (6') are binding and the only possible non-binding constraint is (4'). Therefore, there are 5 binding constraints to solve for 5 unknowns ( $I_m$ ,  $R_m$ ,  $R_f(s_L)$ ,  $R_f(s_H)$ ,  $A$ ). The equilibrium solution is given by:

$$R_f(s_L) = R - \frac{C}{\Delta p}, R_f(s_H) = R - \frac{C}{\Delta p}, h=0, R_m = \frac{C}{\Delta p}, R_u = 0, I_m = \frac{P_B s_L C}{r^* F \Delta p}, I_u = 0$$

The minimum net worth requirement is:

$$A_B(r^*, q) = I - \frac{P_B s_L C}{r^* F \Delta p}$$

and total expected profits when  $A = A_B$  are:  $E[\Pi_{TOT}] = P_G (R - \frac{C}{\Delta p})$

**Proof of Lemma 1:**

Under fixed exchange rates then  $q=0$  and the net worth requirements for being hedged and being not hedged are equal, that is,  $\underline{A}_H = \underline{A}_{NH} < \underline{A}_{SNH}$  and  $\bar{A}_H = \bar{A}_{NH} < \bar{A}_{SNH}$ . Firms with net worth  $A$  such that  $A \geq \underline{A}_{SNH}$  or  $A \geq \bar{A}_{SNH}$  need not hedge. Therefore the relevant region of net worth requirement for which a firm could hedge are  $\underline{A}_H < A < \underline{A}_{SNH}$  if the firm borrow from local banks and  $\bar{A}_H < A < \bar{A}_{SNH}$  if the firm borrows directly in foreign currency. For any net worth  $A$  within these regions profits are given by:

$$E[\Pi_{TOT}] = P_G [(R - R_m - s_L R_u) - \varphi |h|]$$

When  $\varphi \geq 0$ , this expression is strictly lower if the firm is hedged given that  $q \leq \bar{q}$  ensures a positive level of hedging when the firm decides to hedge. Therefore, not hedging is preferred to hedging for firms in the above regions. ■

### Proof of Lemma 2

Assumption 2 guarantees that when hedging is costless ( $\varphi = 0$ ) a hedging firm faces a lower net worth requirement relative to those that decide not to hedge. Then  $\underline{A}_H < \underline{A}_{NH}$  and  $\bar{A}_H < \bar{A}_{NH}$ . Consider a firm with net worth  $\underline{A}_{NH} \leq A < \underline{A}_{SNH}$  within the monitoring region having to choose between hedging or not.

If the firm decides not to hedge then  $R_m = \frac{C}{\Delta p(1-q)}$  and  $R_u = \frac{r^* I_u}{P_G(1-q)}$ , both of

which are lower than  $R_m = \frac{C}{\Delta p}$  and  $R_u = \frac{r^* I_u}{P_G}$  respectively for any  $q > 0$  when the firm

hedges. Therefore, profits are given by:

$$E[\Pi_{TOT}]^H = P_G \left( R - \frac{C}{\Delta p} - \frac{F}{P_G} r^* I_u \right) \text{ if hedged and}$$

$$E[\Pi_{TOT}]^{NH} = P_G \left[ R(1-q) - \frac{C}{\Delta p} - \frac{s_L}{P_G} r^* I_u \right] \text{ if not hedged}$$

Notice that a pair of similar expressions can be obtained for firms with  $\bar{A}_{NH} \leq A < \bar{A}_{SNH}$  borrowing directly from foreign banks (the only difference is that  $R_m=0$  so that  $C/\Delta p$  does not appear in the profit expressions). Assumption 2 guarantees that  $Et[\Pi_{TOT}]^H > Et[\Pi_{TOT}]^{NH}$  and therefore whenever  $\varphi = 0$  the net worth requirement  $\underline{A}_{NH}$  is not relevant and hedging is preferred over not hedging, when not hedging implies insolvency in state H. In cases with  $\underline{A}_{NH} \geq \underline{A}_{SNH}$  and  $\bar{A}_{NH} \geq \bar{A}_{SNH}$  the net worth requirement is also irrelevant because any firm with  $A \geq \underline{A}_{NH}$  or  $A \geq \bar{A}_{NH}$  is solvent enough and need not hedge. ■

### Proof of Proposition 1:

(To be added)

### Proof of Proposition 2:

(To be added)