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Fragmentation across Cones

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ABSTRACT

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This paper examines the effects of fragmentation across cones of diversification in the Heckscher-Ohlin model of international trade. Fragmentation is defined as the splitting of production processes into parts that can be done in different countries. Such fragmentation may occur in a world of factor price equalization (FPE) only if it is costless, and even then it is uninteresting. It becomes more important in a world without FPE, where countries operate in different diversification cones. In that case even costly fragmentation (which uses more resources than the original) may be able to produce a good at a lower cost than the original unfragmented technology, if it can take advantage of different factor prices in different countries. The paper shows when this will be the case, then goes on to examine the effects of fragmentation on factor prices. It is already known that introduction of fragmentation can lead to FPE when FPE did not obtain initially. But it need not do so, and the paper explores the directions of the effects on relative factor prices when they do not become equalized. It turns out that factor prices may actually be driven further apart by fragmentation. This is suggested diagrammatically, and also shown more formally for the case of Cobb-Douglas preferences and technologies.

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I. Introduction

The subject is still fragmentation, as it was in an earlier paper motivated by the previous conference in this series.[1] This time I want to explore further how fragmentation may matter for the prices of factors. This question is most interesting in the context of a Heckscher-Ohlin trade model in which factor prices are *not* equalized by trade, especially if this is due to the countries having sufficiently different factor endowments to make factor price equalization (FPE) impossible.[2] In such a world, with a large number of both goods and countries (but only two factors, to keep things manageable), goods and countries fall into two or more cones of specialization.[3] Within these cones, countries share identical factor prices, but between cones they do not. Fragmentation either will not occur, or is not particularly interesting, within these cones. Therefore I will look primarily at fragmentation across cones.

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[1] See Deardorff (1998).

[2] See Deardorff (1994) for clarification of just how factor endowments must differ in order for FPE to be impossible.

[3] See Deardorff (1979).

In Section II I begin by examining the circumstances under which a new fragmentation technology will be used. There are no surprises here, and the purpose is mainly just to clarify the context of the analysis. A fragmentation technology will be used, under free trade, only if the cost savings from producing fragments in countries with different factor prices are large enough to offset any need for additional resources. In the two-factor model, a relatively simple geometric construction can determine whether or not this is the case. The section also briefly considers the role of tariffs, which can both prevent and stimulate fragmentation depending on whether and where tariffs are imposed on intermediate and final goods.

Sections III and IV provide results that may be more unexpected. Both ask how fragmentation will affect factor prices. In Deardorff (1998) I noted that fragmentation in some sense increases the likelihood of FPE, and indeed it is quite possible that fragmentation will in fact bring factor prices across countries closer together, and even equalize them. However, in a world of many goods, the introduction of a single fragmentation technology is unlikely to accomplish that. The question then becomes whether fragmentation necessarily brings factor prices closer together across countries, and the answer is no. Instead, the direction of change of relative factor prices in each cone depends systematically on how the factor proportions of fragments compare to the average factor intensities within the cones where the fragments are produced.

The analysis in Section III is entirely graphical, and it is also to some extent impressionistic, since the graphs do not really enable one to nail down exactly what factor prices will be after conditions change. In Section IV, therefore, I examine the same issue more explicitly, but in the context of a special case of the model in which all preferences and technologies are Cobb-Douglas. In this context it is feasible to express factor prices under free trade in terms of the intensities of use of the factors in producing industries. From that it is possible to show more explicitly that relative factor prices can be pushed further apart by fragmentation.

II. When Does Fragmentation Happen?

Consider two countries, or groups of countries, with different factor prices under free trade. Unit isocost lines for both countries are shown in Figure 1 as lines ACD for the labor-abundant country, which I will call South, and BCE for the capital abundant country, North.

As discussed in Deardorff (1979), with free trade we can identify which country will produce each (final) good by finding prices that place the good's unit value isoquant (the isoquant for producing one dollar's worth of that good) just tangent to the outermost, or envelope, of the two isocost lines. As a result, all of the goods that capital-abundant North produces will be more capital intensive than all of the goods that South produces, with the possible but not necessary exception of a good whose unit-value isoquant is tangent to both isocost lines on either side of their intersection at point C.[4] Since if there are many goods most are therefore not produced in both countries, I will consider only goods that are produced in only one. One such is good X, whose unit value isoquant is shown in the figure.

Now suppose that fragmentation of the technology for producing X becomes possible. That is, there now exist two different factor combinations (or sets of them) that together permit production of the same amount of good X given by the isoquant. When will these fragments be used instead of the original technology? I will assume in this section that good X and its fragments constitute a sufficiently small part of the total economies that changing to the fragmented technology will not cause a noticeable change in the factor prices in either country. Such changes will instead be the topic of Section III.

If the fragments of technology together were to use a smaller amount of the factors than the original one, then of course they would necessarily be used somewhere, perhaps both of them in the same country as before. As discussed in Deardorff (1998), that possibility is really a combination of both technological improvement and fragmentation, and I will not consider it here. Instead, I will assume always in this paper that fragmentation uses at least a much resources as the unfragmented technology. Therefore, fragmentation is at best only costless, and in general it will be costly in terms of resources used, if it requires larger amounts of factors. I will consider both cases in this section.

Costless Fragmentation

Consider, then, costless fragmentation. When and where will it be used? Suppose in Figure 1 that the same amount of good X produced by the isoquant $X=1/p_x$ can now also be produced using one fragment that requires the vector of factors shown as OZ and another fragment that requires the vector ZY.[5] These are constructed in this case to

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[4] For there to be more than one such good there would have to be factor-intensity reversals, which I assume here do not exist.

[5] For simplicity I will take the fragments to be Leontief fixed-coefficient technologies. If substitution is possible, that can only make the use of the fragments more attractive. However it somewhat complicates

add to exactly the same factors that were being used before in South, where good X was produced. Therefore producers in South could switch to the fragmented technology and it would do them no harm, but it would also do them no good.

But note that of the fragments drawn, one, OZ, would not have been produced in South if it had been a final good, since its capital-labor ratio is above the cutoff given by the intersection of the two isocost lines at C.[6] Therefore we expect that if this particular fragmentation opportunity arises, fragment OZ will be used in North, not South.

That is indeed the case, and costs will fall as a result, as can be verified by the rest of the construction in Figure 1. First draw a new pair of isocost lines, A'C'D' and B'C'E', parallel to ACD and BCE respectively, but both contracted inward toward the origin by the same proportion, and such that the outermost of the shrunken pair passes through the tip of the arrow, OZ. Since the original isocost lines showed the combinations of factors in the two countries worth one dollar, these new ones show the combinations worth some other amount, in this case the cost of producing the fragment OZ in North.[7] Since A'C'D' is an isocost line, all other points on it cost the same in North, including point C', which therefore represents the cost of producing fragment OZ in both North and South. If we now add the other fragment, ZY, to this, we get a bundle of factors that costs the same, in South, as the combined fragmented technology when part of it is used in North. That is, draw vector C'Y' equal in length and direction to ZY,

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the analysis of where a fragment will be produced, since the relevant vectors in each country are those that minimize costs at the country's factor prices.

[6] That is, for the output of the fragment OZ to be produced in South, its price would have to place a unit value isoquant corresponding to OZ tangent to ACD. But at that price, anyone producing it in North would earn a positive profit. Hence, the equilibrium (zero-profit) price for such a fragment is lower, and it would not be produced in South without protection.

[7] If shrinking the isocost lines to OZ had hit OZ in segment A'C', then this would be the cost of producing the fragment in South.

starting at point C'. The tip of this arrow at Y' shows a bundle of factors that costs the same in South as the total fragmented technology, so long as the more capital-intensive fragment is produced in North, where the relative price of capital is lower.

Therefore, the fact that point Y' lies inside isocost line ACD tells us that use of the fragmented technology, even though it is just costless in the sense of using exactly the same total factors as the original technology, actually reduces cost if the fragments are produced in different countries. It also shows us the size of the cost savings, which is measured by the distance of Y' inside ACD.

Costly Fragmentation

Now consider costly fragmentation: a fragmentation technology that uses more combined factors than could have produced the good before. Such a case is shown in Figure 2, where the fragmented technology is again shown by vectors OZ and ZY. This time, however, the two together extend beyond the X isoquant, requiring more total factors. The same construction as in Figure 1 now yields a point Y' that is also beyond the isocost line ACD. Thus, even though in this case one of the fragments could be more cheaply produced in the other country, doing so would not save enough to make up for the greater total factor usage of the fragmented technology.

Therefore, if fragmentation is costly, it may not be used at all, even though one of the fragments could be done more cheaply in the other country. What matters is the size of the cost saving from exploiting different factor prices (the savings seen in Figure 1) compared to the cost of the extra resources required.

Figure 3 shows a case of costly fragmentation that *is* cost-saving if done in different countries. The construction is the same as in Figures 1 and 2; it just turns out differently. Of course I have made it do so by changing two things about the shapes and positions of the curves compared to Figure 2. I have made the capital-intensity of the fragment OZ greater, so that moving it to North could save more cost, and I have reduced the length of the vector ZY, reducing the real cost of the new technology in terms of real resources.

These several figures give us a complete picture of when fragmentation will be used, starting from free trade, in a world of two factors, many goods, and two cones. There are several directions one could go from this for a more complete understanding of the issue. I will consider just two: multiple cones, because they are straightforward; and trade impediments, because they are important. I suspect that the extension to more than two factors is also straightforward for those who visualize easily in three dimensions, but I will not attempt it here.

Before doing the case of multiple cones, however, I would ask the reader to do a bit of drawing for themselves in Figures 2 and 3. In both cases, move the arrow ZY around in the figure, keeping its base on the kinked line E'C'A' and drawing the locus traced out by its tip, Y. The result is an exact image of E'C'A' itself, transposed up and to the right by the vector ZY. The new curve has its kink at point Y', and it extends to the right and left of Y' in lines that are parallel to ACD and BCE respectively. The difference between the cases of Figures 2 and 3 is simply that this new curve lies wholly outside ACE in Figure 2, while part of it passes inside ACE in Figure 3, where fragmentation reduces cost

This technique can easily be applied to situations with more cones. Suppose, as shown in Figure 4, that there are four cones instead of two. The outer envelope of the four isocost lines is $AC_1C_2C_3E$. Good X is produced in the most labor-abundant cone, and again, a fragmentation technology becomes available that uses factor vectors OZ and ZY. This time, fragment OZ could be done at least cost in the second most capital-abundant cone, two cones away from the original X. This can be seen, as before, by shrinking the envelope $AC_1C_2C_3E$ in toward the origin until it just touches vector OZ, which it does in the C_2C_3 cone. The shrunken envelope is labeled A'E', and I now make an exact copy of it, transposing it northeast by the vector ZY. This forms the curve FG, which also has three kinks corresponding to C_1, C_2 and C_3. From the fact that FG passes inside the envelope $AC_1C_2C_3E$, we can conclude that this fragmentation technology will reduce costs, and therefore that it will be used.

With more than two cones, it is also easily possible that neither fragment of a new technology will be employed in the country that originally produced the good. Such an example is shown in Figure 5. Here there are three cones, and I consider the introduction of fragmentation in production of a good, X, that is produced in the middle cone. The fragments, however, differ sufficiently in capital intensity that one of them, OZ, is most efficiently produced in the most capital-abundant cone, while the other, ZY, belongs in the most labor abundant cone. In the case shown, although the combined fragments use more capital and labor than the original technology for producing X, our earlier construction indicates that the fragmented technology is cheaper if performed in these other cones. That is, point Y' is inside the middle cone's isocost line, BDFH. Therefore, when fragmentation becomes possible, if factor prices remain unchanged the entire X industry will shut down in the middle cone and production of the two fragments will arise in the other cones. Of course, if the X industry is large enough, this will induce changes in factor prices, and it may well be that these changes will permit a portion of the X industry in the middle cone to survive. But the potential for complete elimination of an industry is certainly present.

Trade Barriers

Now consider briefly what happens if there are barriers to trade, such as transport costs and/or tariffs. These permit the price of a good to be higher inside one country than in another, so long as the country with the higher price does not export the good. In the diagrams here, that means that the price of the good in any country that exports it must place its unit value isoquant tangent to the envelope of isocost lines, as before. But price in a country that imports the good, or in one that does not trade it at all because of a trade barrier, will place the unit value isoquant in closer to the origin than that. If the country produces the good itself, then domestic price must be high enough to place its isoquant just tangent to its own isocost line.

What does this do to our story? Not much. Although trade impediments, if sufficiently pervasive, will surely change factor prices, this will make them even more likely to be unequal across countries than if trade were frictionless and free. In any case, we can take those factor prices as given for drawing diagrams like Figures 1-4.

However, several effects of trade barriers can easily be understood without recourse to the figures. First, if there is no barrier to trade in X, but there is a barrier to trade in any intermediate input, call it Z, that accompanies fragmentation, then that barrier may render fragmentation unprofitable even when it would otherwise lower costs, as in Figure 3. There, for example, suppose that fragment OZ produces an intermediate input that must then be combined with the resources in vector ZY to produce good X. While the true cost of the intermediate input produced in the capital-abundant country is the same as factor bundle C', the cost to users of it in the labor-abundant country will be higher than this if there is an import tariff on it. It would not take a very large tariff to raise the cost above that of producing X with the old technology, in which case the tariff would prevent fragmentation from occurring.

Second, if there is a barrier to trade in X itself, but no barrier to trade in the intermediate good Z, then even fragmentation that would not have lowered cost with free trade might still be used. In Figure 2, for example, we saw that the fragmented technology would not be used under free trade. But suppose that the capital-abundant country places a tariff on imports of final good X, and suppose also in this case that it is fragment ZY that produces the intermediate input and fragment OZ that completes the processing to produce good X. The tariff on X will raise its price inside the capital-abundant country, and if large enough it can easily raise it above the cost of first importing the intermediate input and then producing X domestically using the fragmented technology.

These two examples indicate that fragmentation can be both encouraged and discouraged by trade barriers.

III. How Does Fragmentation Affect Factor Prices?

In Deardorff (1998) I pointed out that fragmentation has the potential to bring about factor price equalization. This will happen in a situation of free trade if the factor endowments of countries are sufficiently different to prevent FPE initially, and if costless fragmentation enlarges the sets of feasible factor uses enough to accommodate those differences across countries.[8]

But there is no guarantee that this will happen. On the contrary, if there are many goods and if fragmentation becomes possible in only some of them, then the potential for fragmentation to make any difference at all to the economy will be reduced. Indeed, this was my justification in Section II for considering the possibility that factor prices might not change at all.

Here I look at a middle ground. Suppose that fragmentation is quantitatively important enough to matter for factor prices, but not important enough to equalize them across countries. Then in what direction will factor prices move? The answer might seem obvious, from the fact that a large amount of fragmentation at appropriate factor intensities would equalize factor prices. Then surely, one might think, a smaller amount of fragmentation will bring them closer together. That need not be the case, however.

Let the initial situation, prior to fragmentation, be that shown in Figure 6. Factor prices in North and South with free trade are given, as before, by the unit isocost lines BCE and ACD respectively. Each country (or region) is producing a large number of goods, the unit-value isoquants of which are shown and labeled simply X_1, X_2, etc. In general there could be a good produced in both countries, but I will assume that there is not. Instead, South produces all of the world's demand for goods X_1 to X_S and North produces all of the world's demand for the remaining goods, X_{S+1} to X_N.

Now suppose that fragmentation becomes possible for any one of these goods, say good X_f produced in South. As before, the factor requirements of the fragments are

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[8] If fragmentation is costly, then since it will not be used if factor prices are equal, it cannot cause complete

shown as the vectors OZ and ZY. Since OZ is in the Northern cone, and since I have drawn this fragmentation as costless, we know from Section II that the fragmented technology will be used. Consider now the pressures that use of this technology in the two countries will put upon factor prices. These pressures are suggested by the three heavy arrows in the figure, but they require some explanation.

As North begins to produce fragment OZ, this creates additional demands for both factors in amounts given by vector OZ. These demands are added to the factor demands that the country already has for producing other goods. This will tend to bid up the prices of both factors, pushing the unit isocost line inward toward the origin, so I indicate that with an arrow that points inward in the reverse direction of the OZ vector. How this will affect relative factor prices in North is something that I will consider in a moment.

First, though, consider South. There, since the good whose technology is being fragmented is initially produced there, two things happen. First, production of good X_f ceases, reducing demands for both factors by the amounts that were employed there. Since this by itself would reduce factor prices and push the isocost line outward, I draw the arrow F_X pointing away from the origin in the direction of the capital-labor ratio in industry X_f. Second, new production of the fragment ZY requires factors in the ratio given by its slope, so I draw a line parallel to ZY from the origin and indicate another arrow of pressure on factor prices, F_Y, pointing inward where it crosses South's isocost line.

It seems intuitively clear that these arrows are suggestive of how the isocost lines will shift, and perhaps rotate, as a result of fragmentation. The arrow pointing inward on

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FPE.

North's isocost line, for example, suggests that factor prices will both go up, and since it is more or less in the middle of the range of isoquants previously used in North, this will tend to shift the isocost line in an approximately parallel fashion, not particularly changing its slope. On the other hand, if that arrow had been, say, much further up and to the left on the isocost line, representing a much greater increase in demand for capital compared to labor, it would suggest a greater rise in the price of capital and thus a flattening of the isocost curve, as though it were tilted by the push of the arrow.

Similarly, in the South the outward arrow F_X suggests falling factor prices that, because the arrow is drawn high among the capital intensities of South's industries, will make South's isocost line steeper. At the same time, the inward arrow for the more labor-intensive fragment, F_Y, tends to raise factor prices, and may also reinforce that rotation. For the case shown, then, the three arrows together seem to suggest not much change in relative factor prices in North, accompanied by a rise in the wage-rental ratio in South. To avoid clutter, I have not drawn the resulting new isocost lines in Figure 6, but the reader can add them if interested. In North the new isocost line would be shifted inward from BCE slightly, without much change in slope. In South ACD would be rotated somewhat clockwise, pivoting somewhere low on the segment AC, since there is a net reduction at initial prices in demands for factors in South. Together, in this particular case, the changes seem to take us in the direction of FPE.

But if you accept this argument, then such a move toward factor price equality is not at all assured. It depends crucially on where the heavy arrows happen to be, and thus on the factor intensities both of the fragments and of the original technology. There are many possibilities, including that relative factor prices move in the same direction in both countries and that they both move either together or further apart.

Figure 7 shows a case in which relative factor prices move further apart. In the case shown, it is the most labor-intensive good, X_1, for which a fragmentation technology becomes available, with a capital-intensive fragment, OZ, that can be performed in North at lower cost, although it is then the most labor-intensive process being used there. The more labor-intensive fragment, ZY, is of course even more labor-intensive than X_1, although not by much. Once again, arrows F_Z, F_Y, and F_X show the pressures on unit isocost lines, via factor markets and factor prices, due to the fragmentation. Fragment OZ, being the most labor intensive process in North, increases the wage there more than the rental and makes the isocost line steeper (not shown). Fragment ZY would do the same in South if it were not for the offsetting downward pressure on wages due to the cessation of production of X_1. The latter necessarily involves more resources than the fragment ZY, and therefore can easily dominate it, causing the wage-rental ratio to fall. Therefore this is a case in which fragmentation can cause the relative wage to fall in South and rise in North, the two ratios thus moving further apart.

IV. Factor Price Determination

These conclusions about effects on factor prices may seem somewhat less than certain, since I have not provided a full model of how they are determined, and indeed they are only examples of what might happen. But that they are valid examples can be shown rather simply in the following more explicit version of the model.

Suppose that all preferences are identical and Cobb-Douglas, with β_j the share of each consumer's expenditure on good *j*. Suppose that all production functions are also Cobb-Douglas in factors capital and labor, with α_j the capital share and $1-\alpha_j$ the labor share in industry *j*. Finally, suppose that in the initial equilibrium South produces only goods $J_S=1,\ldots,s$ and North produces only goods $J_N=s+1,\ldots,n$. Let any changes we now introduce be small enough so that both countries continue to produce only these same ranges of goods.

Then the algebra of the Cobb-Douglas case tells us that relative factor prices in each country are given by:

$$\omega_i^0 = \frac{w_i^0}{r_i^0} = \frac{\sum_{j\in J_i}(1-\alpha_j)\beta_j}{\sum_{j\in J_i}\alpha_j\beta_j}\frac{K_i}{L_i} = \frac{1-\overline{\alpha}^i}{\overline{\alpha}^i}\frac{K_i}{L_i} \qquad (1)$$

where w_i, r_i, K_i, L_i are the wage, rental rate, capital stock and labor force of region *i* respectively, and $\overline{\alpha}_i$ is the weighted average of the capital shares, α_j, in the industries operating in country *i*:

$$\overline{\alpha}_i = \frac{\sum_{j\in J_i}\alpha_j\beta_j}{\sum_{j\in J_i}\beta_j}$$

Equation (1) says that the wage-rental ratio in a country depends on its factor endowments and on the relative weighted averages of the labor shares compared to the capital shares in its own production. Given the goods that a country or region produces, the relative returns to factors depend inversely on their endowments. But given their endowments, the relative returns also depend positively on the average factor intensities of their own industries. This is the feature of the model that I will use here. The higher the average labor intensity of the goods that a country produces, for example, the higher the country's relative wage.

Now let an industry, $j' \in J_S$, become costlessly fragmented. This means that it is split into two parts, j_1' and j_2', that together produce good j'. Let these fragments also be Cobb-Douglas as follows: Fragment j_1' produces an intermediate input, *z*, using the production function

$$z = K_{z1}^{\alpha_{z1}} L_{z1}^{1-\alpha_{z1}}$$

and fragment j_2' produces final good j' with production function

$$x_{j'} = \left(K_{z2}^{\alpha_{z2}} L_{z2}^{1-\alpha_{z2}}\right)^{1-\gamma} z^{\gamma}$$

Thus, of the share of expenditure $\beta_{j'}$, that goes to good j', a fraction γ is paid to capital and labor employed in fragment j_1' producing the intermediate good *z*, while the fraction 1-γ is paid to capital and labor in the second fragment, j_2', producing the final good.

Suppose for concreteness that $\alpha_{z1} > \alpha_{j'} > \alpha_{z2}$. Also, consistent with this, let fragment j_1' be produced in North, and fragment j_2' in South, as in the figures. North's new wage rental ratio can be found from (1), adding the appropriate terms into both numerator and denominator for the fragment j_1':

$$\begin{aligned} \omega_N^1 &= \frac{(1-\alpha_{z1})\gamma\beta_{j'} + \sum_{j\in J_N}(1-\alpha_j)\beta_j}{\alpha_{z1}\gamma\beta_{j'} + \sum_{j\in J_N}\alpha_j\beta_j}\frac{K_N}{L_N} \\ &= \omega_N^0 + \left(\overline{\alpha}^N - \alpha_{z1}\right)\Omega_N \end{aligned} \qquad (2)$$

where

$$\Omega_N = \frac{\gamma\beta_{j'}}{\left(\alpha_{z1}\gamma\beta_{j'} + \sum_{j\in J_N}\alpha_j\beta_j\right)\overline{\alpha}^N}\frac{K_N}{L_N} > 0$$

The expression for ω_N^1 in (2) is evidently larger than ω_N^0 if α_{z1} is smaller than $\overline{\alpha}^N$, the weighted average of the α_j's in J_N, as was the case in Figure 7.

Similarly, the new wage-rental ratio in South is found from (1) by adding a term in top and bottom for new fragment j_2', and also subtracting the term for the no-longer-produced good j' itself, since it should no longer be included in the summations:

$$\begin{aligned}\omega_S^1 &= \frac{(1-\alpha_{z2})(1-\gamma)\beta_{j'} - (1-\alpha_{j'})\beta_{j'} + \sum_{j\in J_S}(1-\alpha_j)\beta_j}{\alpha_{z2}(1-\gamma)\beta_{j'} - \alpha_{j'}\beta_{j'} + \sum_{j\in J_S}\alpha_j\beta_j}\frac{K_S}{L_S} \\ &= \frac{-\left[(1-\alpha_{j'})\gamma - \delta\right]\beta_{j'} + \sum_{j\in J_S}(1-\alpha_j)\beta_j}{-\left[\alpha_{j'}\gamma + \delta\right]\beta_{j'} + \sum_{j\in J_S}\alpha_j\beta_j}\frac{K_S}{L_S}\end{aligned}$$

where

$$\delta = (\alpha_{j'} - \alpha_{z2})(1-\gamma) > 0$$

This can be manipulated to yield

$$\omega_S^1 = \omega_S^0 - \left[(\overline{\alpha}^S - \alpha_{j'}) - \frac{\delta}{\gamma}\right]\Omega_S \tag{3}$$

where

$$\Omega_S = \frac{\gamma\beta_{j'}}{\left((1-\gamma)\alpha_{z2}\beta_{j'} + \sum_{j\in J_S - j'}\alpha_j\beta_j\right)\overline{\alpha}^S}\frac{K_S}{L_S} > 0$$

Thus ω_S^1 is less than ω_S^0 if α_{z2} and $\alpha_{j'}$ are sufficiently close together, so that δ is small in (3), and if the original good, j', was less capital intensive than the average industry in South. Again, this was the case in Figure 7.

IV. Conclusion

Fragmentation, as defined here, is a diverse phenomenon. When a production process is split into parts, this can be done in a wide variety of ways, and it matters greatly to the economies involved what form the fragmentation takes. In this paper I have looked at just two issues – whether and where fragmented technologies will be used, and the effects of fragmentation on factor prices. In both cases, the answers depend on the nature of the fragmentation. Most important for the results, in both cases, are the factor intensities of the fragments. Also important for the viability of a fragmented technology is its total use of resources.

For the use of fragmented technologies, the analysis shows that even a costly fragmentation technology, one which uses more resources than the original, may be viable. This is true, however, only if countries lie in different diversification cones and (at least) one of the fragments lies in a different cone than the original, unfragmented technology. In that case, the viability of the fragmented technology depends on a tradeoff between its extra cost in terms of resources and the cost saving that can be achieved by performing fragments in countries where factor prices favor them.

Regarding the effects of fragmentation on factor prices, it is of course possible that fragmentation will equalize them across countries. When this does not happen, however, it need not be the case that factor prices are drawn closer together by fragmentation. Instead, the effects on relative factor prices in the countries where the fragmentation takes place depend fairly systematically on the factor intensities of the fragments, as well as that of the original technology. What matters, however, is how these factor intensities compare to the average intensities of processes in use in each country before fragmentation, not their intensities compared to all goods produced globally.[9]

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[9] Davis (1996) has made a similar point for the effects of trade liberalization in a multi-cone model.

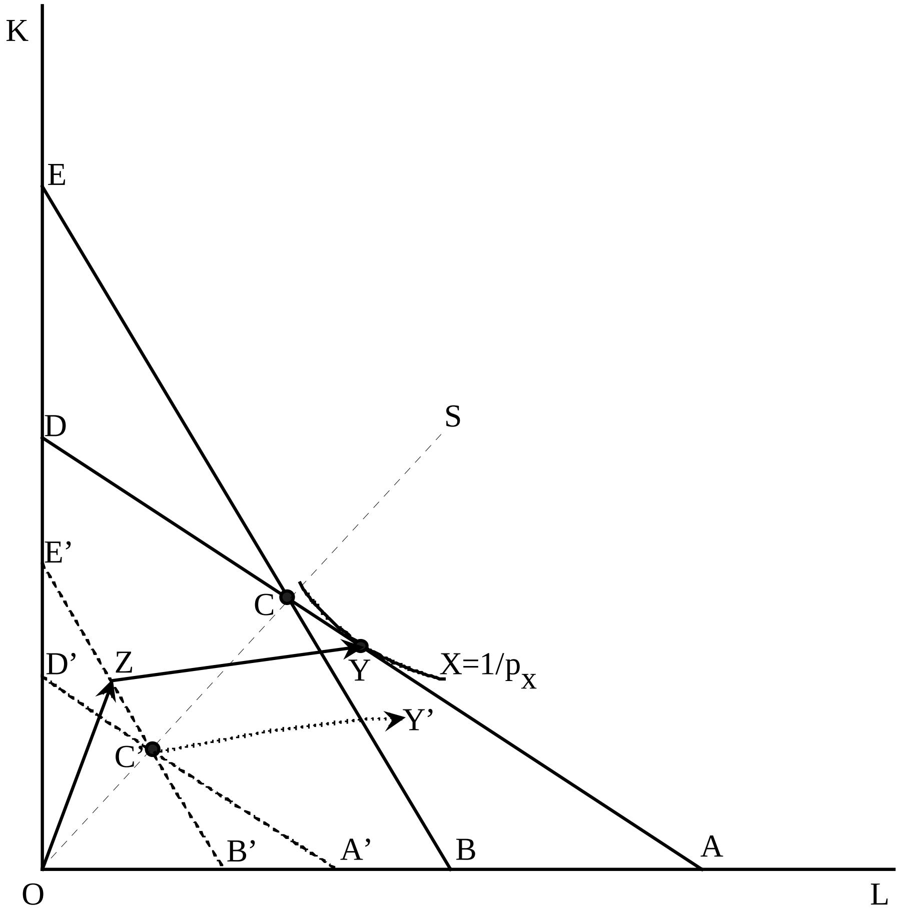

Figure 1:
Cost Saving from Costless Fragmentation

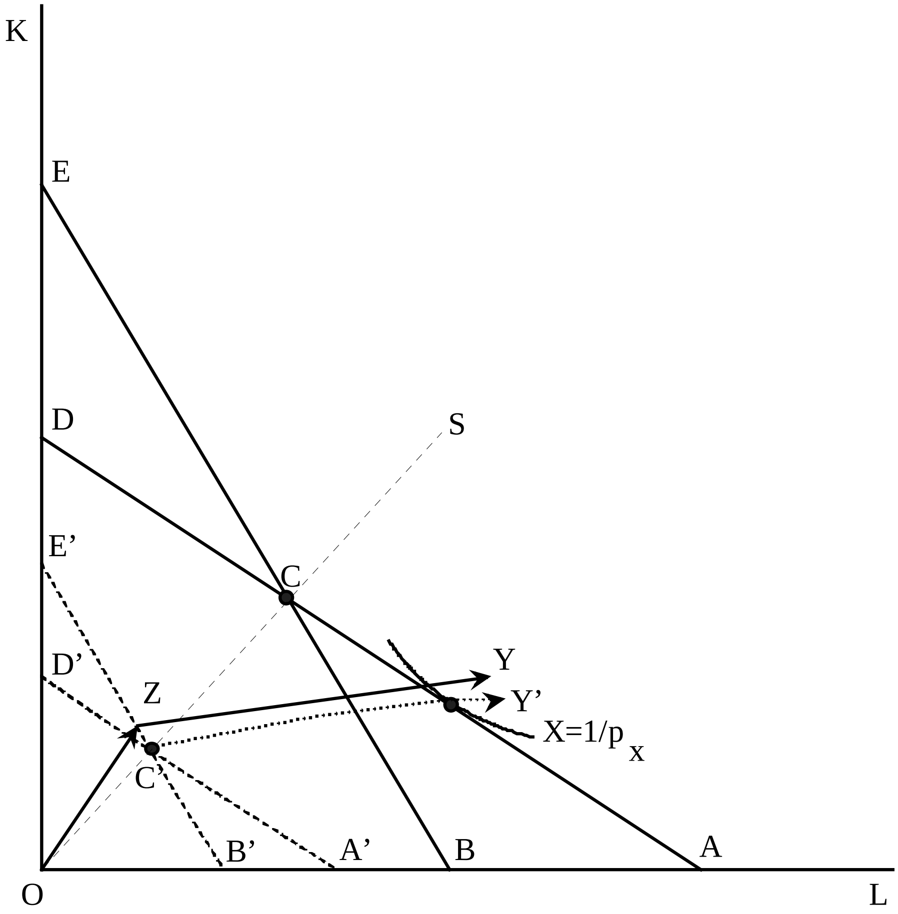

Figure 2:
Cost Increase from Costly Fragmentation

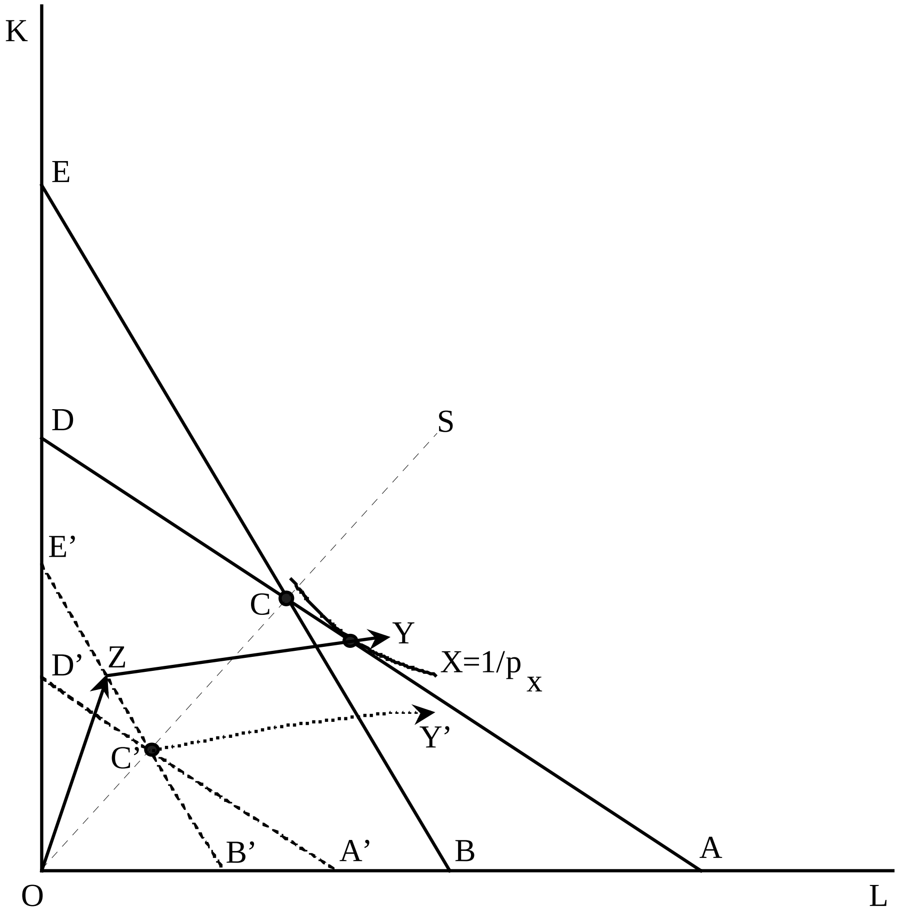

Figure 3:
Cost Saving from Costly Fragmentation

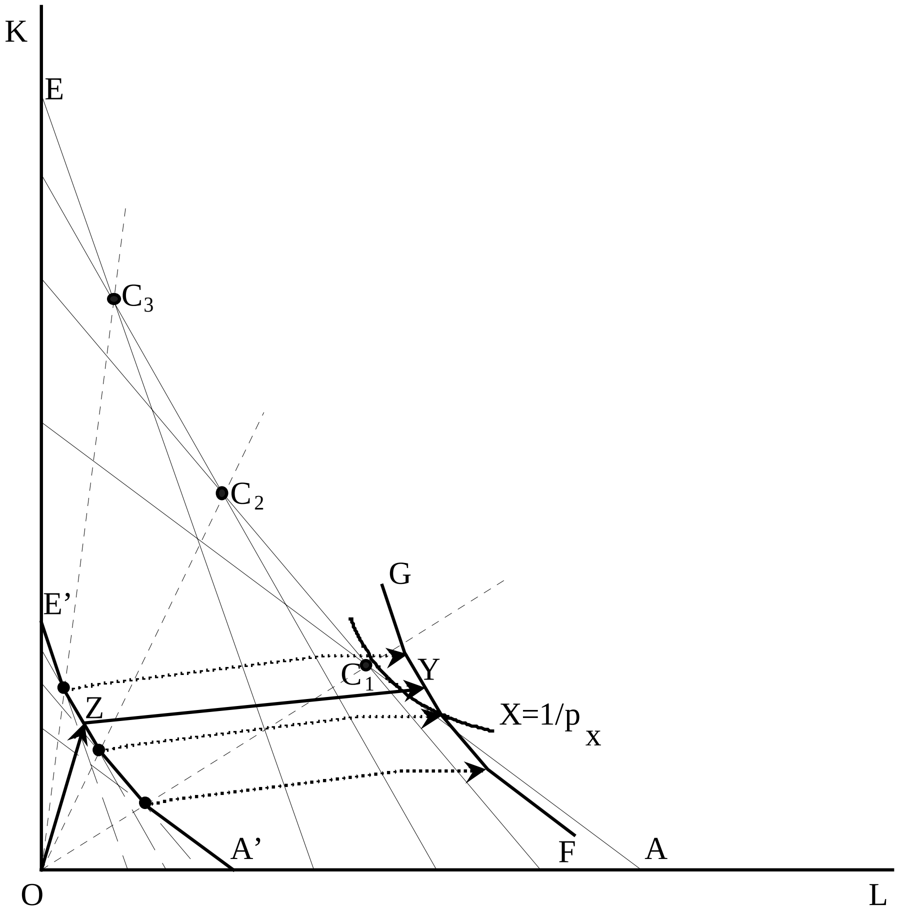

Figure 4:
Cost Saving Fragmentation with Four
Cones of Specialization

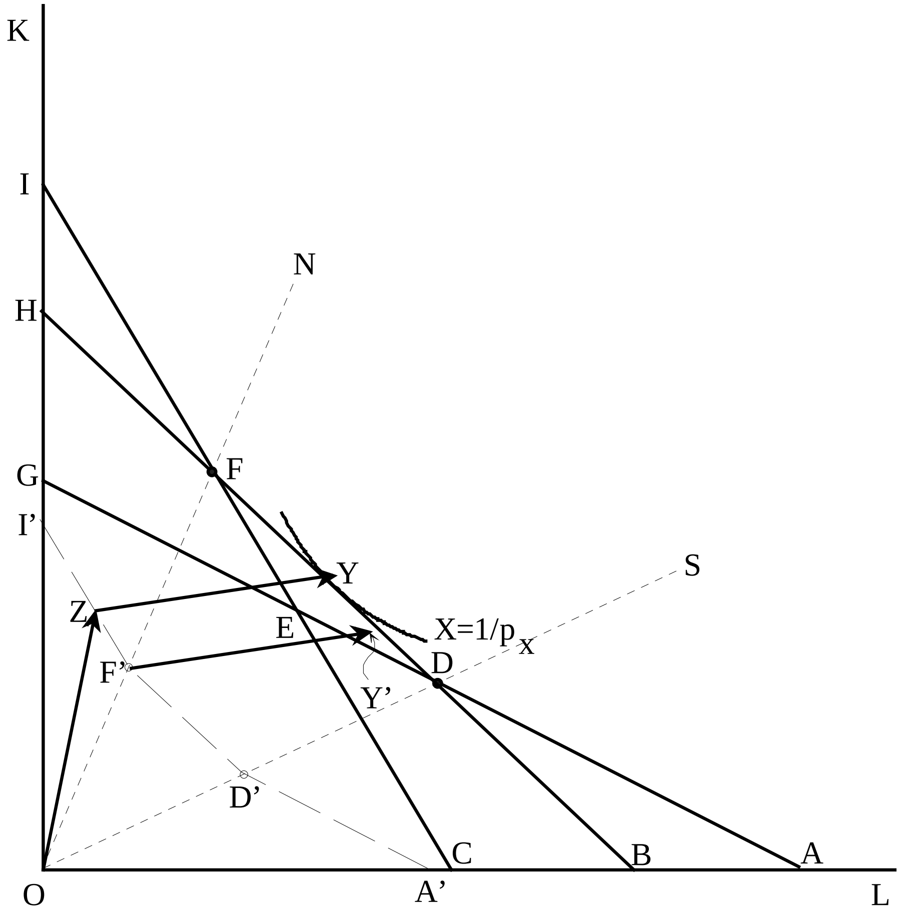

Figure 5:
Fragmentation Off Both Shores

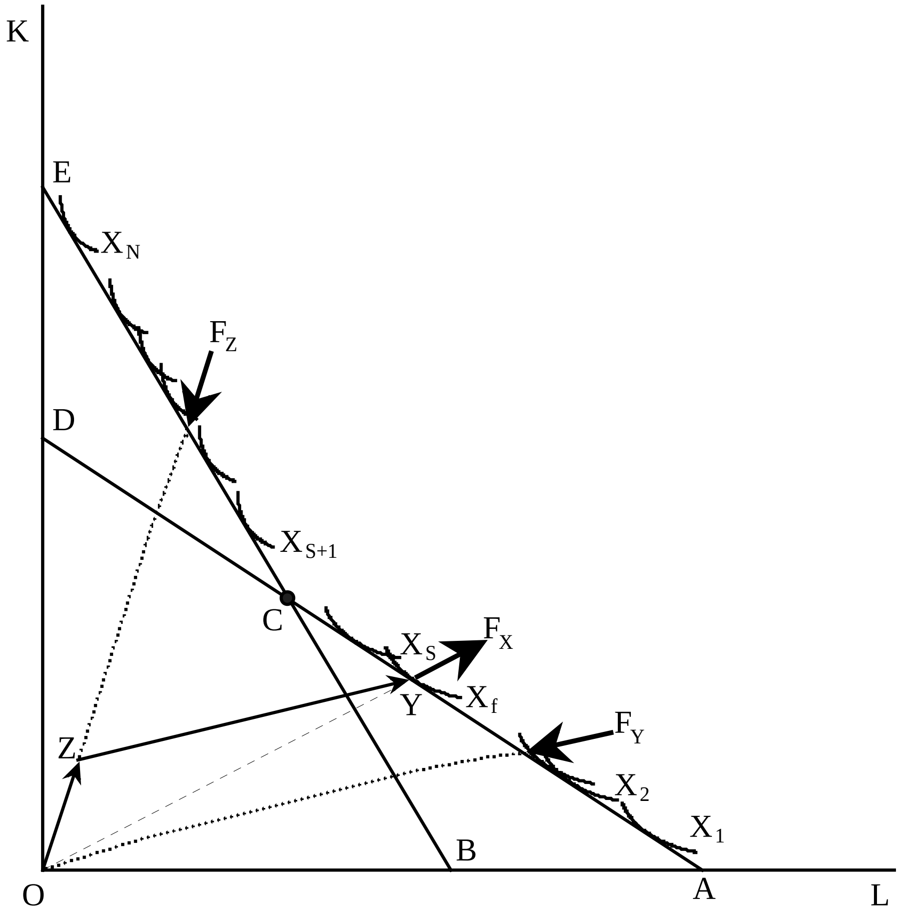

Figure 6:
Fragmentation Pressures on Factor Prices

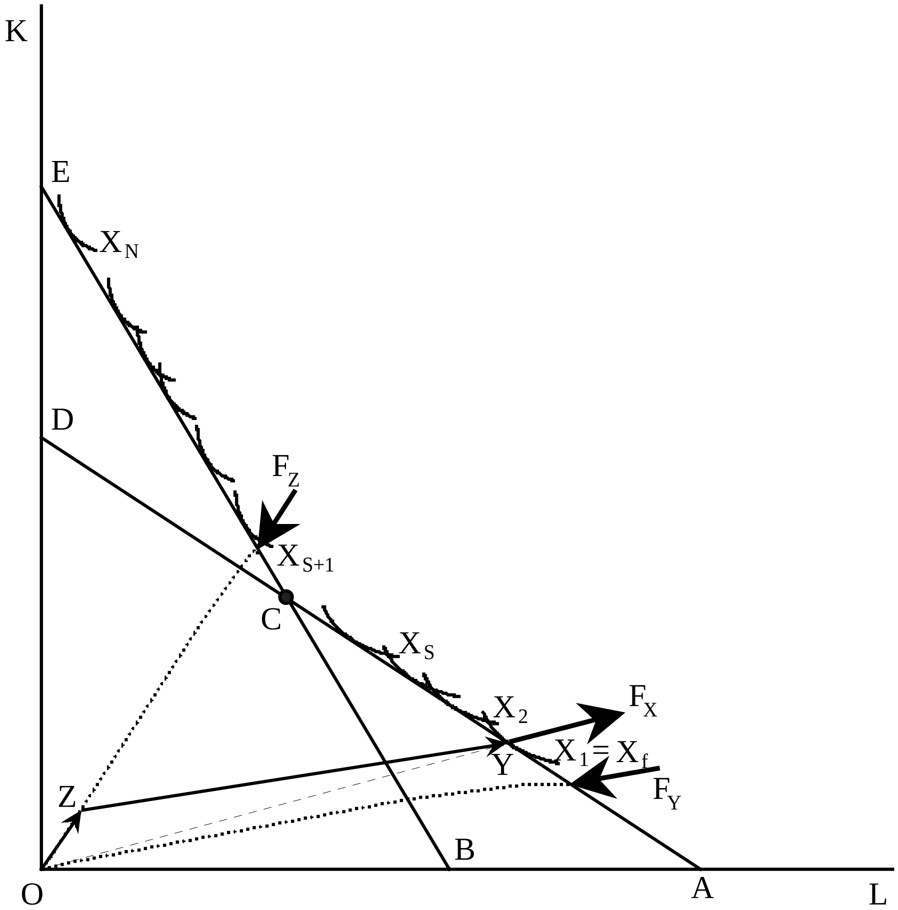

Figure 7:
Fragmentation that Drives Factor Prices Apart