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Tariffs and Production Subsidies as Devices to Relax the Incentive Problem of a Progressive Income Tax System

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Abstract

This paper shows that tariffs and production subsidies can Pareto-improve welfare in a small open economy when a government is concerned with income redistribution under asymmetric information.

In international trade theory, free trade is optimal if the government can use lump-sum taxes and transfers. However, in reality the government cannot use the lump-sum taxes and transfers due to asymmetric information between the government and individuals. In this case, the government needs to use a progressive income tax system for income redistribution. This paper shows that in such a situation even if the government use a Pareto-optimal progressive income tax system under free trade, tariffs can Pareto-improve welfare. That is, tariffs can function as a partial substitute for a Pareto-optimal progressive income tax system under asymmetric information.

1 Introduction

This paper shows that tariffs and production subsidies can Pareto-improve welfare in a small open economy when a government is concerned with income redistribution under asymmetric information.

Since tariffs and production subsidies have a strong effect on income distribution(the Stolper-Samuelson theorem) in a small open economy, it is very tempting for the government to use tariffs and production subsidies for income redistribution. However, the previous literature of optimal taxation theory has negated the use of tariffs or production subsidies for efficient income redistribution. Diamond and Mirrlees(1971) showed that if the government has enough instruments, then efficient production is also optimal for income redistribution. Efficient production in a small open economy implies that the marginal rate of transformation in domestic production must be equal to the international price so long as goods are produced. In other words, the government should not use tariffs, production taxes or production subsidies, and the government should not take advantage of the Stolper-Samuelson effect for income redistribution.(For reference to optimal taxation in an open economy, see Dixit(1985).) However, the previous literature on optimal taxation assumed that information is symmetric between the government and individuals. In this paper, we will show that *production inefficiency* can Pareto-improve welfare when information is asymmetric between the government and individuals.

When information is symmetric between the government and individuals, perfect income redistribution is possible. Since the government can identify individual types, the government can use lump-sum taxes and transfers contingent on individual types. Therefore, free trade is desirable for everyone. However, in reality information is asymmetric between the government and individuals: the government cannot observe and verify whether an individual worker is a skilled or unskilled worker. In this situation, the government is restricted to progressive income taxation as a means to redistribute income. Since income is a function of labor supply, it is very likely that progressive income taxation affects the incentive to supply labor. This paper analyzes such a situation.

Consider a country which is engaged in efficient production(free trade) in a small open economy.[1] Efficient production means that the government does not use tariffs, production taxes or production subsidies which distort the marginal rate of transformation of domestic production away from the international price. Suppose that the government is transferring income from skilled(high-wage) workers to unskilled(low-wage) workers through a progressive income tax system and commodity taxes under asymmetric information. Furthermore, to consider the situation most favorable to efficient production(free trade), suppose that this tax system is Pareto-optimal under efficient production.[2] [3] Then, we wish to ask, " if the government begins to imposes tariffs and adjusts the progressive income tax system and the commodity taxes optimally, then can there be Pareto-improvement to welfare? " [4]

At first, the answer to this question seems to be no. First, according to the Stolper-Samuelson theorem, there are always losers and winners from imposing tariffs. Thus, imposing tariffs does not Pareto-improve welfare if no adjustment is made. Second, as the previous literature of optimal taxation theory suggests, an optimal tax rule generally requires production efficiency. Furthermore, we have assumed that government sets the progressive income tax at the Pareto-optimal level under the condition of efficient production. Third, even if a distortion exists before the tariff is imposed and even if the tariff corrects the distortion, it is not clear how government can distribute the tariff revenue because we have assumed asymmetric information. Fourth, even if production inefficiency corrects the distortion, the tariff also introduces

[1] We prefer to use the term 'efficient production' to 'free trade' because the government can cause the same distortion to an economy without using a tariff as a tariff does by using a consumption tax and a production subsidy.

[2] If this progressive tax system is not Pareto-optimal, then it is self-evident that there is a policy to Pareto-improve welfare. In such a case the problem becomes trivial.

[3] Pareto-optimality with efficient production means that there can is no other policies which improve the welfare of one type of workers without hurting some other types of workers under efficient production.

[4] One might wonder why the government does not provide a production subsidy instead of the tariffs. However, as long as the commodity tax is optimally adjusted, the effect of a tariff with commodity tax adjustment is equal to the effect of a production subsidy. Even if there is no adjustment of the commodity tax, the effect of the tariff without commodity tax adjustment is, at first order, equal to the effect of the production subsidy as long as the tariff is infinitesimal.

distortion in consumption.

However, the answer to the question is 'yes' if the government is redistributing income enough and, as a result, incentive problems exist. Thus, the tariffs function as a partial substitute for a Pareto-optimal progressive income tax system in a small open economy under asymmetric information.

The intuition of this result is the following. Suppose that the government is transferring income from skilled(high-wage) workers to unskilled(low-wage) workers and begins to impose a tariff on a good that uses unskilled labor intensively. According to the Stolper-Samuelson theorem, this tariff increases the real wage of low-wage workers and decreases the real wage of high-wage workers. Initially, the welfare of low-wage workers increases and the welfare of high-wage worker decreases. Thus, the tariff does not Pareto-improve welfare. On the other hand, since the wage ratio between high-wage workers and low-wage workers decreases, the government needs less income transfer. Since the government need less income transfer, the government can re-design the tax system so that the disincentive effect on labor supply is mitigated. As a result, low-wage workers are willing to work more and earn more. Since low-wage workers earn more and need less subsidies, the tax burden of high-wage workers is reduced. Furthermore, the value of the tax reduction exceeds the decrease of the wage of high-wage workers. In addition, since the distortion for consumption decision is of second order, it can be ignored as long as the increase of the tariff is small. In net, high-wage workers are better off by this tariff, and the welfare of both types of workers is improved. This result has an important implication for public policy because the question above approximates the situation which the government faces under free trade. If a country moves from autarky to free trade, there are always winners and loser according to the Stolper-Samuelson theorem. However, since free trade is beneficial in net, the government will redistribute income from winner to losers and will use a progressive income tax system for this purpose. If the government uses a progressive income tax system, then government will have to think whether additionally imposing a tariff or a production subsidy improves welfare. Since imposing a tariff Pareto-improves welfare, this tariff would be supported by everyone. Thus, it is very likely that this policy is pursued politically.

In addition, the above implication suggests an important future research. In the existing literature of numerical evaluation of trade policies, most papers have ignored the domestic tax policies, especially income tax policies. However, the present paper suggests that an income tax structure critically matters for the evaluation of trade policies. It is possible that trade liberalization makes everyone worse off even in a small open economy, for example, if the trade of unskilled-labor intensive goods is liberalized.

Empirically, our result has interesting implications. For example, Magee(1994) tested the Stolper-Samuelson theorem by investigating the Congressional decision on trade. He found that the groups representing different factor owners tend to take the same position regarding the trade policies and concluded that the Stolper-Samuelson theorem does not hold empirically. However, in our model the groups representing different factor owners take the same position regarding the trade policies because such trade policies Pareto-improve welfare.

Another prediction is about the sharp relationship between domestic tax policies and trade polices. Since the previous literature shows that efficient production is optimal, it does not predict any particular pattern of trade intervention. However, this paper predicts that government will subsidizes the sector which uses intensively workers to whom the government redistributes income.

2 Relation to Previous Literature

There is a large literature which discusses the relationship between domestic tax policies and trade policies. A consensus in the literature is that if the government has enough of the instruments of tax policies, then it is optimal to keep production efficient. This implies that the government should not use either tariffs, production taxes or production subsidies in a small open economy.

In the international trade literature, Dixit and Norman(1980,1986) shows that a free trade equilibrium is Pareto-superior to an autarky equilibrium even without lump-sum transfers. Their idea is that the gain from trade can be redistributed by using commodity taxes and taxes contingent on factors.[5] The problem of Dixit and Norman(1980) is that they require the government to have enough information about workers. For example, if factors in an economy consist of skilled workers and unskilled workers, then taxation contingent on factors implies that the government uses different tax rates on different skill levels. Since the government usually cannot observe and verify skill levels of workers due to asymmetric information, those factor taxes would not be feasible in reality.

In international trade theory, many papers have been written on the second best policy in a small open economy; e.g., Bhagwati(1971). From the points of view of those literature, it is not surprising that tariffs can improve welfare. However, most papers in the previous literature ignored both information asymmetry and distributional effects by assuming a representative worker or through the use of lump-sum taxes and transfers. Although some papers do not assume the use of the lump-sum taxes and transfers, they required prefect information between the government and workers. See, e.g., Diamond and Mirrlees(1971). The present paper takes a different approach. Here it is assumed that there are two types of workers and information is asymmetric between the government and workers; the government cannot identify whether a worker is skilled or unskilled. Under such a circumstance, it is not obvious that a tariff can increase the welfare of both types of workers even if it can Pareto-improve welfare under the regime of perfect income transfer, because the government would not know whether the recipient of the tariff revenue is a skilled or unskilled worker.

In the public finance literature, Diamond and Mirrlees(1971) proved that if the government can use commodity taxes and perfect contingent factor taxes, then production efficiency is optimal even when the government is concerned with income redistribution. Since the international

[5] Dixit and Norman(1980) claimed that the gain from trade can be redistributed without lump-sum transfers by using commodity taxes and taxes contingent on factors. Furthermore, Dixit and Norman(1986) claimed that such a commodity taxes and taxes contingent on factors are incentive compatible. We agree with the first claim completely. However, the second claim seem to be strong and seems to depends on the real economic circumstances.

price is another production possibility frontier in a small open economy , the result of Diamond and Mirrlees implies that the government should keep the marginal rate of transformation equal to the international price. [6] However, the same criticism to Dixit and Norman(1985) can be applied to the result of Diamond and Mirrlees because they assumed perfect information. Mirrlees(1971) studied the problem of optimal progressive income taxation under asymmetric information. In his seminal paper, Mirrlees(1971) studied the optimal income tax system when the government cannot identify individual characteristics due to asymmetric information between the government and workers. By using Mirrlees' framework, Atkinson and Stiglitz(1976, 1980) examined whether the government should use commodity taxes additionally when the government uses a progressive income tax system to redistribute income. They showed that if the government provides a progressive income tax system and the utility function is weakly separable between goods and leisure, then imposing commodity taxes is not optimal for income redistribution. Stiglitz(1982) clarified the essential points of the work of Mirrlees in an economy where there are two types of workers.

The contribution of the present paper is to extend Stiglitz(1982) and to incorporate the optimal taxation model under asymmetric information into the general equilibrium framework. By combining the general equilibrium framework and asymmetric information, this paper reveals an interesting relationship between trade policies and tax policies.

3 The Structure of The Model

Given our interest in the relationship between efficient income redistribution and trade policies in a small open economy, it is necessary to modify some standard assumptions in international trade theory. First, we assume that factor supply is elastic because we are concerned with incentive problems of income redistribution and inelastic factor supply eliminates incentive problems. Second, since we want to consider the case where perfect income transfer is not

[6] For detailed explanation, see Dixit(1985).

possible, we need to assume that the government cannot identify individual types. [7] If the government can identify whether an individual is a skilled or unskilled worker, then perfect income transfer is possible by lump-sum taxes. Thus, it is necessary to assume that the government cannot identify individual types.

The basic model is an extension of Stiglitz(1982). There are two agent types, two goods and two factors in a small open economy. Normalizing the international price of good 1 to one, the international price of good 2 is p^*. Following Gordon and Levinsohn(1990), we assume that there are no commodity tax or tariff on good 1 without loss of generality. Let t be the commodity tax and σ be the tariff(or an export subsidy) on good 2. Then

$$p = p^* + \sigma$$

$$q = p + t = p^* + \sigma + t$$

where p is the producer's price for good 2 and q is the consumer's price for good 2.

In this economy, there are two types of workers, skilled workers and unskilled workers. Skilled workers, s, supply skilled labor, and unskilled workers, u, supply unskilled labor. We assume that the utility function of a worker of type i $\{i = s, u\}$ is weakly separable between consumption goods and labor and strictly quasi-concave with respect to (c_1, c_2, l):

$$V(c_1, c_2, l) = V(U(c_1, c_2), l),$$

where l is labor supply and (c_1, c_2) is consumption of good 1 and 2. [8] It is clear from the assumption of weak separability between goods and labor that the choice between c_1 and c_2

[7] Those two assumption are standard in the literature of optimal progressive income tax.

[8] We use the assumption of weak separability to compare with the previous results in the public finance literature. The main result of the present paper holds without the assumption of weak separability. However, this assumption clarifies the essential points and make the geometrical analysis possible. Weak separability is a key assumption in the optimal taxation literature in public finance. The literature of optimal taxation asserts that if a utility function is weakly separable between goods and labor and the government uses a progressive income tax, then uniform commodity tax is optimal under the assumption of constant marginal cost of production. Atkinson and Stiglitz(1980) justified the assumption of weak separability by noting that most functional forms used in empirical studies assume weak separability.

is independent of l if individual income is given. Thus, we can define the sub-indirect utility function $v(q, x)$

$$v(q,x) \equiv \max_{\{c_1,c_2\}} U(c_1, c_2)$$
$$\text{subject to} \qquad c_1 + q * c_2 \leq x,$$

where q is the consumer price of good 2 and x is income of the worker. The sub-indirect utility function is the maximized value from consumption of the two goods when income, x, and price, q, are given. The indirect utility function with labor supply, l, of the type i worker is

$$V^i(v(q,x), l). \tag{1}$$

Let wage be, w_s and w_u, for skilled and unskilled types, respectively. We assume that the government observes only total income and that the government cannot observe workers' types. Therefore, in order to provide a Pareto-efficient progressive income tax system, the government needs to present a tax schedule. However, since there are only two types of workers in this economy, at most only two points on the tax schedule will be selected by workers. Therefore, we do not lose any generality from focusing on only those two points. Invoking 'the Revelation principle,' we assume that the government presents a menu of tax 'contracts ' so that individual workers each self-select the contract which the government intended. [9] Let $T(\cdot)$ be a tax or subsidy function and T_i be a tax or subsidy when the government observes total income, $w_i * l^i$. Then,

$$T_i = T(w_i * l^i).$$

Let x^i be the net income of worker i when the government observes total income $w_i * l^i$;

$$x^i \equiv w_i * l^i - T_i = w_i * l^i - T(w_i * l^i).$$

[9] Some might contend that this type of tax system is not observed at all. However, ' the Revelation Principle' proves that any tax system, or any mechanism, can be replicated by an incentive compatible direct mechanism. Since real resource allocation is completely replicated by the incentive compatible direct mechanism, we do not lose any generality by assuming this type of tax system. For more detailed explanation about 'the Revelation Principle', see Gibbons(1992) and Myerson(1985).

Then, incentive compatible constraints can be written as [10]

$$V^s(v(q,x^s),l^s) \geq V^s(v(q,x^u),\frac{w_u * l^u}{w_s}) \tag{2}$$

$$V^u(v(q,x^u),l^u) \geq V(v(q,x^s),\frac{w_s * l^s}{w_u}). \tag{3}$$

The first constraint means that a skilled worker has an incentive to work l^s, report income $w_s * l^s$ truthfully and receive net income x^s instead of mimicking an unskilled worker, working $\frac{w_u * l^u}{w_s}$, reporting income $w_u * l^u$ and receiving net income x^u. The second constraint implies that an unskilled worker has an incentive to work l^u , to report income $w_u * l^u$ and to receive net income x^u truthfully instead of mimicking a skilled worker, working $\frac{w_s * l^s}{w_u}$, reporting $w_s * l^s$ and receiving net income x^s. If the government provides an incentive compatible menu $\{(l^u, x^u),$ $(l^s, x^s)\}$, then all workers reveal their preferences truthfully and choose the 'contracts' which the government intended.

Then, the objective of the government is to choose an incentive compatible menu $\{(l^u, x_u), (l^s, x_s)\}$ and to provide Pareto-efficient income tax system subject to its budget constraint. The government budget constraint is

$$T_s + T_u + t * (c_2^s + c_2^u) + \sigma * m_2 \geq 0, \tag{4}$$

where m_2 is the amount of imports or exports of good 2. The first two terms are the revenue from a progressive income tax and the third term is the tax revenue from a commodity tax. The fourth term is the revenue from the tariff on good 2 or the expenditure to the export subsidy on good 2. In this formulation, we ignore the problem of government expenditure to simplify the problem. Therefore, taxation is only for income redistribution. By using the definition of x^s and x^u, we can rewrite the budget constraint in the following way:

$$w_s * l^s + w_u * l^u + t * (c_2^s + c_2^u) + \sigma * m_2 - x^s - x^u \geq 0.$$

Next, we need to specify production. There are two industries F_1 and F_2. Each industry exhibits constant returns to scale and the technology is convex. Industry produces good 1 and

[10] We allow the possibility that workers pay a negative tax; i.e., the government pays a subsidy to workers.

industry produces good 2. We assume that each industry uses skilled and unskilled labor and produces output y_1 and y_2, respectively. Thus,

$$y_1 = F^1(l_1^s, l_1^u) \qquad y_2 = F^2(l_2^s, l_2^u). \tag{5}$$

Given the price of w_s, w_u and p, each industry maximizes its profit. We assume that industry 2 is always unskilled labor intensive.[11]

From the Learner-Pearce box diagram, we can calculate the range of the diversification cone. Let $\underline{\gamma}$ be a lower bound of the diversification cone and let $\overline{\gamma}$ be an upper bound of the diversification cone. Then, production is diversified if

$$\underline{\gamma} \ll \frac{l^u}{l^s} \ll \overline{\gamma}.$$

Production is specialized in the sector employing unskilled labor intensively if

$$\overline{\gamma} \leq \frac{l^u}{l^s}.$$

Production is specialized in the sector employing skilled labor intensively if

$$\frac{l^u}{l^s} \leq \underline{\gamma}.$$

It is clear that $\underline{\gamma}$ and $\overline{\gamma}$ are functions of the tariff level. Thus, we can write $\underline{\gamma}$ and $\overline{\gamma}$ as:

$$\underline{\gamma} \equiv \underline{\gamma}(\sigma) \qquad \overline{\gamma} \equiv \overline{\gamma}(\sigma). \tag{6}$$

Let $C_k(w_s, w_u)$ be the cost function for production of one unit of good k $\{k = 1, 2\}$. If production is diversified, then wages are determined by the following two equations:

$$C_1(w_s, w_u) = 1 \qquad C_2(w_s, w_u) = p^* + \sigma \tag{7}$$
$$\text{if} \qquad \underline{\gamma}(\sigma) \ll \frac{l^u}{l^s} \ll \overline{\gamma}(\sigma)$$

[11]Assuming industry 2 is unskilled labor intensive is arbitrary.

If production is specialized in the unskilled-labor intensive sector, then w_s and w_u are determined as follows:

$$w_s = p * \frac{\partial F^2(l^s, l^u)}{\partial l^s} \qquad w_u = p * \frac{\partial F^2(l^s, l^u)}{\partial l^u} \tag{8}$$
$$\text{if} \qquad \frac{l^u}{l^s} \leq \underline{\gamma}(\sigma)$$

If production is specialized in the skilled-labor intensive sector, then wages are determined as follows:

$$w_s = \frac{\partial F^1(l^s, l^u)}{\partial l^s} \qquad w_u = \frac{\partial F^1(l^s, l^u)}{\partial l^u} \tag{9}$$
$$\text{if} \qquad \overline{\gamma}(\sigma) \leq \frac{l^u}{l^s}$$

Using equation (7), (8) and (9), we can calculate a function that maps from the ratio of skilled and unskilled labor supply to the wage ratio:

$$\frac{w_u}{w_s} \equiv \Omega(\frac{l^u}{l^s}; \sigma). \tag{10}$$

Figure 1 shows this function. From the Stolper-Samuelson theorem, if production is diversified

$$\frac{\partial w_u}{\partial \sigma} \gg 0 \qquad \frac{\partial w_s}{\partial \sigma} \ll 0 \qquad \frac{\partial \Omega}{\partial \sigma} \gg 0. \tag{11}$$

Next, we need to specify the relationship among the outputs, tariff and factor inputs. Given the labor supply of skilled and unskilled workers and equilibrium conditions in labor market, a production possibility frontier is uniquely determined. Thus, we can write y_2 as a function of l^s, l^u and $p^* + \sigma$:

$$y_2 = Y_2(l^s, l^u, p^* + \sigma).$$

From the Rybczynski theorem, if the supply of unskilled labor increases , then output which uses intensively unskilled labor increases given fixed prices. Thus,

$$\frac{\partial Y_2(l^s, l^u, p^* + \sigma)}{\partial l^u} \gg 0.$$

If the supply of skilled labor increases, then output of good 2 decreases.

$$\frac{\partial Y_2(l^s, l^u, p^* + \sigma)}{\partial l^s} \ll 0.$$

Since the production possibility frontier is convex set given the amount of l^s and l^u,

$$\frac{\partial Y_2(l^s, l^u, p^* + \sigma)}{\partial (p^* + \sigma)} \gg 0.$$

Finally, we need to specify equilibrium conditions. We can write c_2^s and c_2^u as functions of p and x^i:

$$c_2^i \equiv D(q, x^i).$$

Goods market equilibrium implies

$$c_1 \equiv c_1^s + c_1^u = y_1 + m_1 \qquad c_2 \equiv c_2^s + c_2^u = y_2 + m_2,$$

where m_k is the amount of imports or exports of good $k \quad \{k = 1, 2\}$. Balance of trade implies

$$m_1 + p^* * m_2 = 0.$$

Walras law guarantees that one of goods market equilibrium conditions is automatically met. Using the equilibrium condition of second goods market, we can rewrite the government budget constraint as

$$w_s(\sigma, \frac{l^u}{l^s}) * l^s + w_u(\sigma, \frac{l^u}{l^s}) * l^u + (t + \sigma) * \{D(p^* + t + \sigma, x^s) + D(p^* + t + \sigma, x^u)\}$$
$$-\sigma * y_2(p^* + \sigma, l^s, l^u) - x^s - x^u \geq 0.$$

4 Optimization by The Government

This section examines whether increasing a tariff from zero can Pareto-improve welfare when the government is redistributing income by the Pareto-optimal progressive income tax and the commodity tax under efficient production(free trade). In order to examine the effect of the tariff, we first characterize the Pareto-optimal progressive income tax and the commodity tax under efficient production. The Pareto-optimal progressive income tax with the commodity tax for a given level of the tariff is obtained by solving the following problem:

Main Program

$$\max_{\{x^s,x^u,l^s,l^u,t\}} V^s(v(p*+t+\sigma,x^s),l^s)$$

subject to

$$V^u(v(p^*+t+\sigma,x^u),l^u))\geq \bar{U}^u \qquad \text{(MUC)}$$

$$V^s(v(p^*+t+\sigma,x^s),l^s))\geq V^s(v(p^*+t+\sigma,x^u),\Omega(\sigma,\frac{l^u}{l^s})*l^u) \qquad \text{(ICS)}$$

$$V^u(v(p^*+t+\sigma,x^u),l^u)\geq V^u(v(p^*+t+\sigma,x^s),\frac{l^s}{\Omega(\sigma,\frac{l^u}{l^s})}) \qquad \text{(ICU)}$$

$$w_s(\sigma,\frac{l^u}{l^s})*l^s+w_u(\sigma,\frac{l^u}{l^s})*l^u+(t+\sigma)*(D(p^*+t+\sigma,x^s)+D(p^*+t+\sigma,x^u))$$
$$-\sigma*y_2(p^*+\sigma,l^s,l^u)-x^s-x^u\geq 0 \qquad \text{(BC)}$$

σ is given.

The first constraint (MUC) addresses the exogenous minimum utility level of unskilled workers. $\bar{U}^u$ is the level of minimum utility determined by an exogenous factor.[12] The Pareto-efficiency requires that the utility of the skilled worker be maximized for a utility level of unskilled workers. ICS is an incentive compatibility constraint arising from the progressive income tax system for skilled workers, and ICU is an incentive compatibility constraint for unskilled workers. BC is the government budget constraint. In the analysis below, we assume that production is diversified in interior points when $\sigma=0$. Thus, $\frac{\partial\Omega}{\partial l^s}=0$ and $\frac{\partial\Omega}{\partial l^u}=0$ at $\sigma=0$.

Let μ, λ_1, λ_2 and λ_3 be the Lagrangian multiplier of MU, ICS, ICU and BC, respectively. Then, the Lagrangian $L(l^s,l^u,x^s,x^u;\sigma)$ for a given level of the tariff is

[12] One obvious choice of $\bar{U}^u$ would be the level of the utility of unskilled workers in the autarky equilibrium. However, the results of this paper do hold with greater generality.

$$
\begin{aligned}
&L(l^s, l^u, x^s, x^u; \sigma) \\
&\equiv V^s(v(p * + t + \sigma, x^s), l^s) + \mu\{V^u(v(p^* + t + \sigma, x^u), l^u)) - \bar{U}^u\} \\
&+\lambda_1\{V^s(v(p^* + t + \sigma, x^s), l^s)) - V^s(v(p^* + t + \sigma, x^u), \Omega(\sigma, \frac{l^u}{l^s}) * l^u)\} \\
&+\lambda_2\{V^u(v(p^* + t + \sigma, x^u), l^u) - V^u(v(p^* + t + \sigma, x^s), \frac{l^s}{\Omega(\sigma, \frac{l^u}{l^s})})\} \\
&+\lambda_3\{w_s(\sigma, \frac{l^u}{l^s}) * l^s + w_u(\sigma, \frac{l^u}{l^s}) * l^u + (t + \sigma) * (D(p^* + t + \sigma, x^s) + D(p^* + t + \sigma, x^u)) \\
&-\sigma * y_2(p^* + \sigma, l^s, l^u) - x^s - x^u\} \qquad (12)
\end{aligned}
$$

For convenience, we define

$$
\begin{aligned}
&\frac{\partial V^i(v(q, x^i), l^i)}{\partial v(q, x^i)} \equiv V_1^{ii}, \qquad \frac{\partial V^i(v(q, x^i), l^i)}{\partial l^i} \equiv V_2^{ii} \\
&\frac{\partial V^i(v(q, x^j), \frac{w_j * l^j}{w^i})}{\partial v(q, x^j)} \equiv V_1^{ij} \quad (i \neq j), \qquad \frac{\partial V^i(v(q, x^j), \frac{w_j * l^j}{w^i})}{\partial \frac{w_j * l^j}{w^i}} \equiv V_2^{ij} \quad (i \neq j) \\
&\frac{\partial v(q, x^i)}{\partial q} \equiv v_q(q, x^i), \qquad \frac{\partial v(q, x^i)}{\partial x^i} \equiv v_x(q, x^i) \\
&\frac{\partial D(q, x^i)}{\partial q} \equiv D_q(q, x^i), \qquad \frac{\partial D(q, x^i)}{\partial x^i} \equiv D_x(q, x^i) \\
&\text{i, j=s, u} \\
&\frac{\partial \Omega}{\partial \sigma} \equiv \Omega_\sigma
\end{aligned}
$$

Then, the first order conditions for l^s, l^u, x^s, x^u and t under efficient production(free trade) are

$$
\begin{aligned}
&l^s: \quad \left.\frac{\partial L}{\partial l^s}\right|_{\sigma=0} = V_2^{ss} + \lambda_1 V_2^{ss} - \lambda_2 V_2^{us} * \frac{1}{\Omega} \\
&+\lambda_3 w_s = 0 \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
&x^s: \quad \left.\frac{\partial L}{\partial x^s}\right|_{\sigma=0} = V_1^{ss} * v_x(p^* + t, x^s) + \lambda_1 * V_1^{ss} v_x(p^* + t, x^s) \\
&-\lambda_2 V_1^{us} * v_x(p^* + t, x^s) + \lambda_3 * t * D_x(p^* + t, x^s) - \lambda_3 = 0 \qquad (14)
\end{aligned}
$$

$$l^u: \quad \left.\frac{\partial L}{\partial l^u}\right|_{\sigma=0} = \mu V_2^{uu} - \lambda_1 V_2^{su} * \Omega(p) + \lambda_2 V_2^{uu} + \lambda_3 w_u = 0 \tag{15}$$

$$x^u: \quad \left.\frac{\partial L}{\partial x^u}\right|_{\sigma=0} = \mu V_1^{uu} * v_x(p^* + t, x^u) - \lambda_1 V_1^{su} * v_x(p^* + t, x^u)$$
$$+\lambda_2 V_1^{uu} * v_x(p^* + t, x^u) + \lambda_3 * t * D_x(p^* + t, x^u) - \lambda_3 = 0 \tag{16}$$

$$t: \quad \left.\frac{\partial L}{\partial t}\right|_{\sigma=0} = V_1^{ss} v_q(p^* + t, x^s) + \mu V_1^{uu} v_q(p^* + t, x^u)$$
$$+\lambda_1\{V_1^{ss} v_q(p^* + t, x^s) - V_1^{su} v_q(p^* + t, x^u)\}$$
$$+\lambda_2\{V_1^{uu} v_q(p^* + t, x^u) - V_1^{us} v_q(p^* + t, x^s)\}$$
$$+\lambda_3\{D(p^* + t, x^s) + D(p^* + t, x^u)$$
$$+t * D_q(p^* + t, x^s) + t * D_q(p^* + t, x^u)\} = 0 \tag{17}$$

First, we characterize the optimal commodity tax under efficient production(free trade). By using the first order conditions (14), (16),the Roy's identity about sub-indirect utility function and the Slutsky equation, equation (17) becomes(see Appendix)

$$\lambda_3 * t * \{\frac{\partial h(p^* + t, v^s)}{\partial q} + \frac{\partial h(p^* + t, v^u)}{\partial q}\} = 0 \tag{18}$$

where $h(p^* + t, v^i)$ is the Hicksian demand function and $v^i = v(p + t, x^i)$. Since λ_3 is the marginal utility of the government budget, it is strictly positive. From the assumption of a strictly quasi-concave utility function, the partial derivative of the Hicksian demand with respect to price is strictly negative. This implies $t = 0$. Thus, the government should not use a commodity tax for income redistribution when the government provides a Pareto-optimal progressive income tax system under efficient production(free trade) and the utility function is weakly separable.

Result 1 *If the government uses a Pareto-optimal progressive income tax system and if the utility function is weakly separable between goods and labor, then the government should not use commodity taxes for income redistribution under efficient production(free trade).*

This is a restatement of the well-known result in the public finance literature that if the marginal cost of production is constant, then non-uniform commodity taxes should not be used for income redistribution in a closed economy under a progressive income tax system. For reference, see Atkinson and Stiglitz(1976,1980), Laffont and Tirole(1993), Christiansen(1984) and Konishi(1995). However, note that Result 1 does not hold if the utility function is not weakly separable. As we will show in the Appendix, if the utility function is not weakly separable, then commodity taxes are generally not zero and the sign of the commodity tax depends on the complementarity of the good with leisure.

Next, we examine the first order conditions for l^s, l^u, x^s and x^u. Those first order conditions show that $\mu \gg 0$. However, as for λ_1 and λ_2, the values of λ_1 and λ_2 depend on the value of $\bar{U}^u$. There are three possible cases to be considered.

1. Redistributive case: $\lambda_1 \gg 0$ and $\lambda_2 = 0$.

   This case is the focus of most of the optimal progressive income tax literature and is called 'normal case' by Stiglitz(1982). When $\lambda_1 \gg 0$ and $\lambda_2 = 0$, ICS is binding and ICU is not binding. Therefore, high-wage workers mimic low-wage workers, but low-wage workers do not mimic high-wage workers because the government redistribute income from high-wage workers to low-wage workers sufficiently. In this case, high-wage workers pay taxes and low-wage workers receive subsidies. This case is more likely if the government sets $\bar{U}^u$ at relatively high level.

2. Regressive case: $\lambda_1 = 0$ and $\lambda_2 \gg 0$.

   When $\lambda_1 = 0$ and $\lambda_2 \gg 0$, ICU is binding and ICU is not binding. Thus, low-wage workers mimic high wage workers but high-wage workers do not mimic low-wage workers because the government transfers income from low-wage workers to high-wage workers. In this case, low-wage workers pay taxes and high-wage workers receive subsidies. This case is more likely if the government sets $\bar{U}^u$ at a medium relatively lower level.

3. First best case: $\lambda_1 = 0$ and $\lambda_2 = 0$.

In this case neither ICS nor ICU is binding. From the *Main Program*, it is clear that the solution in this case coincides with the Pareto-efficient allocation under the regime of perfect income transfer because the problem under the regime of perfect income transfer is to maximize objective function under the constraints of only MU and BC. Therefore, the allocation in this case becomes the first best allocation . For example, from equation (13), (14),(15) and (16),

$$\frac{V_2^{ss}}{V_1^{ss}} = -w_s \qquad \frac{V_2^{uu}}{V_2^{uu}} = -w_u \tag{19}$$

Thus, the marginal rate of substitution is equal to wage rate for both types of workers. This case is more likely if the government sets $\bar{U}^u$ at the medium level and when the government's income transfer is relatively small. [13]

4.1 The Effect of A Tariff

This subsection will show that if the government increases the tariff from zero and adjusts other policies optimally, then welfare is Pareto-improved. To check for the robustness, it is useful to consider two cases: the case where the commodity tax is optimally adjusted, and the case where the commodity tax is fixed at zero. The first case measures the effect of a production distortion and the second case measures the effect of both a consumption distortion and a production distortion. Let $\{x^s(\sigma), l^s(\sigma), x^u(\sigma), l^u(\sigma), t(\sigma)\}$ be the solution of the *Main Program*. The utility of skilled workers from this allocation is $V^s(v(p^* + \sigma + t(\sigma), x^s(\sigma)), l^s(\sigma))$. Then , the first case is measured by $\left.\frac{d\,V^s(v(p^*+\sigma+t(\sigma),x^s(\sigma)),l^s(\sigma))}{d\,\sigma}\right|_{\sigma=0}$ and the second case is measured by $\left.\frac{d\,V^s(v(p^*+\sigma,x^s(\sigma)),l^s(\sigma))}{d\,\sigma}\right|_{\sigma=0}$. Although two effects are conceptually different, the envelope theorem shows that both effects are equal(see Figure 2) and are calculated as follows:

$$\left.\frac{d\,V^s(v(p^* + \sigma, x^s(\sigma)), l^s(\sigma))}{d\,\sigma}\right|_{\sigma=0} = \left.\frac{d\,V^s(v(p^* + \sigma, x^s(\sigma)), l^s(\sigma))}{d\,\sigma}\right|_{\sigma=0} = \left.\frac{\partial L}{\partial \sigma}\right|_{\sigma=0}.$$

[13] It is easy to check that the case where $\lambda_1 \gg 0$ and $\lambda_2 \gg 0$ does not occur when the utility function is weakly separable.

Then, from the equation (12) , the effect of increasing a tariff from zero is

$$\left.\frac{\partial L}{\partial \sigma}\right|_{\sigma=0} = V_1^{ss} v_q(p^* + t + \sigma, x^s) + \mu V_1^{uu} v_q(p^* + t + \sigma, x^u)$$
$$\lambda_1 \{V_1^{ss} v_q(p^* + t + \sigma, x^s) - V_1^{su} v_q(p^* + t + \sigma, x^u)\}$$
$$\lambda_2 \{V_1^{uu} v_q(p^* + t + \sigma, x^u) - V_1^{us} v_q(p^* + t + \sigma, x^s)\}$$
$$+\lambda_3 \{\frac{\partial w_s}{\partial \sigma}(\sigma) * l^s + \frac{\partial w_u}{\partial \sigma}(\sigma) * l^u + D(p^* + t + \sigma, x^u) + D(p^* + t + \sigma, x^s)$$
$$-y_2 + t * D_q(p^* + t + \sigma, x^s) + t * D_q(p^* + t + \sigma, x^u)\}$$
$$-\lambda_1 V_2^{su} * \Omega_\sigma * l^u$$
$$-\lambda_2 V_2^{us} \{-\frac{\Omega_\sigma}{\Omega^2}\} * l^s \tag{20}$$

By using the Roy's identity, equation (14) and equation (16), equation(20) becomes

$$\left.\frac{\partial L}{\partial \sigma}\right|_{\sigma=0} = -c_2^s * \lambda_3 - c_2^u * \lambda_3 - \lambda_1 * V_2^{su} \Omega_\sigma * l^u - \lambda_2 * V_2^{us} \frac{-\Omega_\sigma}{\Omega^2} * l^s$$
$$+\lambda_3 * \{\frac{\partial w_s}{\partial \sigma} * l^s + \frac{\partial w_u}{\partial \sigma} * l^u + D(p^*, x^s) + D(p^*, x^u) - y_2\} \quad . \tag{21}$$

From perfect competition and the envelope theorem(see Appendix 2),

$$\frac{\partial w_s}{\partial \sigma}(\sigma) * l^s + \frac{\partial w_u}{\partial \sigma}(\sigma) * l^u = y_2 \quad . \tag{22}$$

Therefore, equation (21) can be rewritten

$$\left.\frac{\partial L}{\partial \sigma}\right|_{\sigma=0} = -\lambda_1 * V_2^{su} \Omega_\sigma * l^u - \lambda_2 * V_2^{us} \frac{-\Omega_\sigma}{\Omega^2} * l^s \quad .$$

The above equation indicates that the effect of increasing the tariff from zero critically depends on the structure of the progressive tax system under free trade because λ_1 and λ_2 show the structure of the progressive income tax system under free trade. There are again three possible cases that need to be considered:

1. Redistributive case: $\lambda_1 \gg 0$ and $\lambda_2 = 0$

   When λ_1 is strictly greater than zero and λ_2 is equal to zero, then the progressive income tax system is 'redistributive' under efficient production. In this case the effect of increasing the tariff from zero is

$$\left.\frac{\partial L}{\partial \sigma}\right|_{\sigma=0} = -\lambda_1 * V_2^{su}\Omega_\sigma * l^u \gg 0 \tag{23}$$

Thus, imposing a tariff or an export subsidy on a low-wage labor intensive good Pareto-improves welfare. In equation (23), λ_1 is the shadow price of the incentive compatibility constraint of high-wage workers. When the government redistributes income from high-wage workers to the low-wage workers, high-wage workers have an incentive to mimic low-wage workers to work less. In order to avoid the situation where the high-wage workers mimic low-wage workers, the government needs to impose some constraints on the progressive tax system. λ_1 measures the (shadow) cost of one of such constraints.

2. Regressive case. $\lambda_1 = 0$ and $\lambda_2 \gg 0$

   When λ_1 is equal to zero and λ_2 is strictly greater than zero, then the progressive income tax is 'regressive' under efficient production. In this case, the effect of increasing the tariff from zero is

   $$\left.\frac{\partial L}{\partial \sigma}\right|_{\sigma=0} = \lambda_2 * V_2^{us}\frac{\Omega_\sigma}{\Omega^2} * l^s \ll 0$$

   Therefore, imposing a tariff on high-wage labor intensive good Pareto-improves welfare.

3. First best case. $\lambda_1 = 0$ and $\lambda_2 = 0$

   In this case, $\left.\frac{\partial L}{\partial \sigma}\right|_{\sigma=0} = 0$. This does not mean that the tariff does not change the welfare when the progressive income tax system is the first best. It only means that the first order effect of the tariff on welfare is zero because when the allocation is the first best, $\sigma = 0$ is the optimal level of the tariff. Figure 3 illustrate this situation.

From the examination of three cases, we obtain the following proposition.

Proposition 1 (Stolper-Samuelson Theorem under A Progressive Income Tax System) *Consider a country which is engaged in free trade(efficient production) in a small open economy* *and whose production is diversified. Suppose the government of the country is using a Pareto-optimal progressive income tax under free trade and the tax system is 'redistributive.' Then, imposing a tariff(or providing an export subsidy) on a low-wage labor intensive good improves welfare of both high-wage and low-wage workers. If the income tax system is 'regressive', then imposing a tariff (or providing an export subsidy) on a high-wage labor intensive good improves welfare of both types of workers.*

The main idea of this proposition is that the government can Pareto-improve welfare by imposing a tariff or an export subsidy on a good which intensively uses workers to whom the government transfers income. Thus, tariffs or export subsidies function as a partial substitute for a Pareto-optimal progressive income tax system.

This result seems to contradict to the conclusion of the Stolper-Samuelson theorem in international trade theory. The Stolper-Samuelson theorem says that if the government imposes a tariff on an unskilled labor intensive good, then the welfare of unskilled labor increases and the welfare of skilled labor decreases. However, our proposition asserts that if income is redistributed by a progressive income tax system, then imposing a tariff benefits everyone because it reduces the incentive problem of the tax system and makes the tax system more efficient.

The intuition of this proposition is as follows. Suppose that the government is redistributing income from high-wage workers to low-wage workers. The incentive problem causes low-wage workers to supply less labor than is optimal. Suppose that the government begins to impose a tariff marginally on a low-wage labor intensive good. Then, by the Stolper-Samuelson theorem, the real wage of low-wage workers increases and the real wage of high-wage workers decreases. Thus, the tariff does not Pareto-improve welfare initially. On the other hand, since the wage differential becomes smaller, the government need less income transfer from high-wage workers to low-wage workers. Then, the government can redesign the progressive income tax system so that the disincentive effect on labor supply is mitigated. As a result, low-wage workers become willing to work more, earn more and need less transfer. Thus, the tax burden of high-wage workers is reduced. Furthermore, the reduction of the tax burden offsets the decrease of the wage of high-skilled workers completely. Thus, the utility level of high-wage workers becomes better off and the tariff on the low-wage intensive good Pareto-improves welfare.

One question to the above argument is why the reduction of the tax burden of high-wage workers is enough to compensate for the effect of the decrease of the real wage of high-wage workers. The basic reason is that to relax the incentive compatibility constraint has the first order effect while the distortion by the tariff has only the second order effect. Figure 4 will demonstrate the basic mechanism. In Figure 4, we analyze the effect of a production subsidy instead of a tariff because the essential economic mechanism comes from production distortion and it also makes the geometric analysis possible. In Figure 4, an unskilled worker chooses the point P and a skilled worker chooses point X and the progressive income tax system is 'redistributive'. PQ is the amount of the subsidy to the unskilled worker and YX is the amount of the tax from the skilled worker. We call the indifference curve from P to R 'the Laffer curve for unskilled workers,' because the government can reduce the amount of the subsidies to the unskilled workers by decreasing the marginal tax rate for them and making them work more with the same utility level. However, since the skilled workers have also an incentive to select any point on PR, the the government cannot implement the point on 'the Laffer curve for unskilled workers.'

Suppose that the government now begins to provide a production subsidy for the good 2. Then, since the wage of the unskilled workers goes up and the wage of the skilled workers goes down, the indifference curve of the unskilled workers shifts down and the indifference curve of the skilled workers shifts up. As a result, the government can decrease the marginal tax rate for the unskilled workers and make them work more and choose the point, P^*, on 'the Laffer curve for unskilled workers.' This change from P to P^* can be decomposed into two effect: from P to P^{**} and from P^{**} to P^* where P^{**} is the intersection between new indifference curve of the unskilled workers and the horizontal axis from the point P. By noting that $PQ - P^{**}Q^{**} = l^u * \frac{\partial w^u}{\partial \sigma}$, the decrease of the subsidies to the unskilled workers is calculated as follows:

$$PQ - P^*Q^* = PQ - P^{**}Q^{**} + P^{**}Q^{**} - P^*Q^* = l^u * \frac{\partial w^u}{\partial \sigma} + \beta,$$

where β is some positive value.

On the other hand, since the wage of the skilled workers decreases, their tax burden needs to be reduced if the government wants to keeping them at least well off. Then, the necessary reduction of the tax burden is

$$YX - Y^*X^* = l^s * \frac{\partial w^s}{\partial \sigma}$$

Finally, the amount of the production subsidy for the good 2 is y_2. Thus, the change of the government revenue is

$$l^s * \frac{\partial w^s}{\partial \sigma} + l^u * \frac{\partial w^u}{\partial \sigma} + \beta - y_2.$$

Since $l^s * \frac{\partial w^s}{\partial \sigma} + l^u * \frac{\partial w^u}{\partial \sigma} = y_2$, the government revenue increases even if the utility of worker is kept constant. This implies that the production subsidy can Pareto-improve welfare.

In the above analysis, we focused on the infinitesimal change of the tariff and the local property of the equilibrium. Suppose that the local maxima is the global maxima and the tax system is 'redistributive.' Then, the first order condition for the *optimal tariff* is

$$-\lambda_1 * V_2^s * \Omega_\sigma * l^u - \sigma * \lambda_3 \frac{\partial y_2}{\partial \sigma} = 0 \tag{24}$$

And, the first order condition for the *optimal commodity tax* is

$$(t + \sigma) * \lambda_3 * \{\frac{\partial h(p^* + t, v^s)}{\partial q} + \frac{\partial h(p^* + t, v^u)}{\partial q}\} = 0 \tag{25}$$

Since the partial derivative of the Hicksian demand function with respect to price is strictly negative from the assumption of substitutability between goods, the above first order condition for the commodity tax implies that $t = -\sigma$. Therefore, when the utility function is weakly separable and if the non-zero optimal tariff is chosen under a progressive income tax system, then the commodity tax is chosen so that the commodity tax offsets the effect of the tariff on consumption distortion.[14]

Result 2 *If the government uses a Pareto-optimal progressive income tax system and a tariff and if the utility function is weakly separable, then the optimal commodity tax is used to offset the consumption distortion of the tariff.*

In the above analysis, we did not specify the level of $\bar{U}^u$. Therefore, Proposition 1 does not state whether the government restricts trade or enhances trade. In another word, Proposition 1 does not say whether the government imposes a tariff or provides an export subsidy. If we additionally assume that the government maintain the level of the utility of workers at least at the autarky equilibrium level, then the following proposition is obtained.

Proposition 2 (Pattern of Trade Intervention) *If the government maintains the level of utility of workers at least at the autarky equilibrium level, then the Pareto-optimal tax system restricts trade and imposes a tariff when the amount of income redistribution is large.*

From the Stolper-Samuelson theorem, if a country moves from an autarky equilibrium to free trade, then the factor which is used intensively in import-competing sector is hurt and the factor which is used intensively in exporting sector gains. Thus, the government transfers

[14]Readers might argue that, from the equation (25), the distortion of consumption goods is not necessary for the Pareto-optimal allocation, and what is necessary for Pareto-optimal allocation is the production distortion. Although this argument is true when the utility function is weakly separable between goods and labor, it is not generally true when the utility function is not weakly separable. When the government implements the progressive income tax and, as a result, the disincentive effect on labor supply exists, it is possible to mitigate this disincentive effect by imposing commodity taxes on the goods which are complement to leisure, although it is not possible to eliminate disincentive effect completely. By reducing the consumption of the goods which are complement to leisure, the government can increase the labor supply. Thus, it is optimal to use the commodity tax even under free trade when the government use a progressive income tax system and the utility function is not weakly separable. In Appendix , we will show that both the consumption distortion and the production distortion are optimal in equilibrium if goods are complement or substitute to leisure and if the government uses the progressive income tax to redistribute income.

income from the latter to the former to maintain the level of the utility of workers at the autarky equilibrium level. In this case, as proposition 1 shows, imposing a tariff(not providing an export subsidy) Pareto-improves welfare.

5 Discussion

It would be useful to discuss the limitation of this paper. Obviously, the assumption of two factors plays important role in the present paper. To extend the present model into a model with N goods and N skill factors would be difficult because there are many possible relationship between the change of goods prices and the change of factor prices.

One way to extend the present model is to to increase the number of types of workers while restricting the model into 2*2 economy. For example, if we assume that each skilled worker has a different human capital level and the wage of skilled workers is determined from their efficient human capital level, then Proportion 1 would hold in the multi-types, two factors and two goods economy.

Recently, many papers focus on the 'endogenous tariff formation.' These studies generally model a political process as a game among factor owners and interpret trade policies as a result of the game. One criticism of this research has been the lack of justification for why players of this game would be concerned with trade policies in the first place.[15] It seems more natural that the players of this game are concerned with direct income transfer(tax progressivity) rather than trade policies. This paper provides one answer to this criticism. Even after the bargaining over direct income transfer(tax progressivity) among the players, all players of this game agree to disturb the trade patterns because disturbing the trade patterns Pareto-improves welfare if the government uses a direct income transfer.

[15] See Rodrick(1995) for an nice survey of the literature.

6 Conclusion

About fifty years ago, Stolper-Samuelson found the strong distributive effect of tariffs in a small open economy. In spite of their finding, the optimal taxation literature and the second best literature have been generally negative regarding the use of tariffs as optimal policy instruments for efficient income distribution. This paper showed that tariffs function as the partial substitute of the Pareto-efficient progressive income tax system under asymmetric information in a small open economy.

When the government cannot use the lump-sum tax and transfer due to asymmetric information between the government and workers, incentive problems arise. This paper explicitly models the relationship between income transfer and incentive problems under asymmetric information. The result shows that imposing a tariff Pareto-improves welfare even if the government redistributes income by a Pareto-efficient progressive income tax system under free trade . This is true because imposing a tariff changes the wage ratio between high-wage workers and low-wage workers and relaxes the incentive problems of a progressive income tax system. Since most countries uses a progressive income tax system, our model can provide one answer to the question of why a country uses tariffs.

7 Appendix

7.1 Derivation of equation (18)

From Roy's identity about sub-indirect utility function,

$$v_q(p^* + t, x^i) = -c_2^i * v_x(p^* + t, x^i) \tag{26}$$

Therefore, equation (17) becomes

$$\begin{aligned}
&-c_2^s\{V_1^{ss} * v_x(p^* + t, x^s) + \lambda_1 V_1^{ss} * v_x(p^* + t, x^s) - \lambda_2 V_1^{us} v_x(p^* + t, x^s)\} \\
&-c_2^u\{\mu V_1^{uu} * v_x(p^* + t, x^s) - \lambda_1 V_1^{su} v_x(p^* + t, x^u) + \lambda_2 V_1^{uu} * v_x(p^* + t, x^u)\} \\
&+\lambda_3\{D(p^* + t, x^s) + D(p^* + t, x^u) + t * D_q(p^* + t, x^s) + t * D_q(p^* + t, x^u)\} = 0
\end{aligned} \tag{27}$$

By using equation (14) and (16), the equation becomes

$$-c_2^h\{\lambda_3 - \lambda_3 * t * D_x(p^* + t, x^s)\} - c_2^u\{\lambda_3 - \lambda_3 * t * D_x(p^* + t, x^u)\}$$
$$+\lambda_3 * \{D(p^* + t, x^s) + D(p^* + t, x^u) + t * D_q(p^* + t, x^s) + t * D_q(p^* + t, x^u)\} = 0$$

Since $D(p^* + t, x^i) = c_2^i$, the above equation becomes

$$\lambda_3 * t * \{D_x(p^* + t, x^s) * c_2^s + D_q(p^* + t, x^s)$$
$$+D_x(p^* + t, x^u) * c_2^u + D_q(p^* + t, x^u)\} = 0$$

Slutsky equation about conditional demand is

$$\frac{\partial h(p^* + t, v^i)}{\partial q} = D_x(p^* + t, x^i) * c_2^s + D_q(p^* + t, x^i)$$

where $h(p^* + t, v^i)$ is the Hicksian demand function and $v^i = v(p + t, x^i)$. Thus, we obtain

$$\lambda_3 * t * \{\frac{\partial h(p^* + t, v^s)}{\partial q} + \frac{\partial h(p^* + t, v^u)}{\partial q}\} = 0 \tag{18}$$

7.2 Derivation of equation (22)

Note that a perfect competition implies

$$w_s * l^s + w_u * l^u = y_1 + (p^* + \sigma) * y_2$$

Thus the magnitude of $\frac{\partial w_s}{\partial \sigma}(\sigma) * l^s + \frac{\partial w_u}{\partial \sigma}(\sigma) * l^u$ is equivalent to the change of total revenue under the condition of fixed labor supply. Fixed labor supply determines the shape of production possibility frontier. Therefore, knowing the output of y_1 and y_2 is equivalent to the solving the following problem.

$$\pi(\sigma) \equiv \max_{\{y_1, y_2\}} y_1 + (p^* + \sigma) * y_2$$
$$\text{subject to} \quad (y_1, y_2) \in \bar{Y}(l^s, l^u)$$

where $\bar{Y}(l^s, l^u)$ is production possibility set. From the envelope theorem,

$$\frac{d\pi(\sigma)}{d\sigma} = y_2.$$

Thus, we obtain $\frac{\partial w_s}{\partial \sigma}(\sigma) * l^s + \frac{\partial w_u}{\partial \sigma}(\sigma) * l^u = y_2$.

8 Appendix 2

When the utility function is general forms $U(c_1, c_2, l)$, then the conditional indirect utility function is defined as

$$V^i(q, x^j; l) \equiv \max_{\{c_1^i, c_2^i\}} U^i(c_1^i, c_2^i, l)$$

subject to $\quad c_1^i + q * c_2^i \leq x^j$

l and q are given $\quad$ i, j= s, u

Let $D^i(q, x^j, l)$ be the conditional demand function of good 2 when type i worker supplies l units of labor and she is given x^j income and the price of good 2 is q.

Then, Lagrangian $\bar{L}(l^s, l^u, x^s, x^u; \sigma)$ for this problem is

$$\begin{aligned}
&\bar{L}(l^s, l^u, x^s, x^u; \sigma) \\
&\equiv V^s(p * +t + \sigma, x^s; l^s) + \mu\{V^u(p^* + t + \sigma, x^u); l^u)) - \bar{U}^u\} \\
&+\lambda_1\{V^s(p^* + t + \sigma, x^s; l^s)) - V^s(p^* + t + \sigma, x^u, \Omega(\sigma; \frac{l^u}{l^s}) * l^u)\} \\
&\lambda_2 * \{V^u(p^* + t + \sigma, x^u; l^u) - V^u(p^* + t + \sigma, x^s; \frac{l^s}{\Omega(\sigma, \frac{l^u}{l^s})})\} \\
&\lambda_3 * \{w_s(\sigma, \frac{l^u}{l^s}) * l^s + w_u(\sigma, \frac{l^u}{l^s}) * l^u + (t + \sigma) * (D^s(p^* + t + \sigma, x^s; l^s) + D^u(p^* + t + \sigma, x^u, l^s) \\
&-\sigma * y_2(p^* + \sigma, l^s, l^u) - x^s - x^u\} \qquad (28)
\end{aligned}$$

The first order conditions of l^u,l^s,x^s,x^u and t under efficient production are

$$\left.\frac{\partial \bar{L}}{\partial l^s}\right|_{\sigma=0} = \left.\frac{\partial \bar{L}}{\partial l^u}\right|_{\sigma=0} = \left.\frac{\partial \bar{L}}{\partial x^s}\right|_{\sigma=0} = \left.\frac{\partial \bar{L}}{\partial x^u}\right|_{\sigma=0} = \left.\frac{\partial \bar{L}}{\partial t}\right|_{\sigma=0} = 0$$

First we characterize the first order condition for the commodity tax under free trade. By using the same technique in section 4, the first order condition for the commodity tax, $\left.\frac{\partial \bar{L}}{\partial t}\right|_{\sigma=0} = 0$ becomes

$$\lambda_1 * \frac{\partial V^{su}(q, x^u; \frac{w^u * l^u}{w^s})}{\partial x^u} * \{D^s(q, x^u; \frac{w^u * l^u}{w^s}) - D^u(q, x^u; l^u)\}$$
$$+\lambda_2 * \frac{\partial V^{us}(q, x^s; \frac{w^s * l^s}{w^u})}{\partial x^s} * \{D^u(q, x^s; \frac{w^s * l^s}{w^u}) - D^s(q, x^s; l^s)\}$$
$$+\lambda_3 * t * \left\{\frac{\partial h(p^* + t, V^s; l^s)}{\partial q} + \frac{\partial h(p^* + t, V^u; l^u)}{\partial q}\right\} = 0 \tag{29}$$

where $h(p^* + t, V^i; l^i)$ is the Hicksian demand function with the condition of labor supply and $V^i = V(p^* + t, x^i; l^i)$. If the utility function is weakly separable, then the demand does not depends on the labor supply and the first two terms are zero.Thus, $t = 0$ is optimal. However, when the utility function is not weakly separable, then the sign of the commodity tax depends on the complementarity of the good 2 with labor and the structure of the progressive income tax system.

1. Redistributive case. $\lambda_1 \gg 0$ and $\lambda_2 = 0$.

   In this case, equation (29) becomes

$$\lambda_1 * \frac{\partial V^{su}(q, x^u; \frac{w^u * l^u}{w^s})}{\partial x^u} * \{D^s(q, x^u; \frac{w^u * l^u}{w^s}) - D^u(q, x^u; l^u)\}$$
$$+\lambda_3 * t * \{\frac{\partial h(p^* + t, V^s; l^s)}{\partial q} + \frac{\partial h(p^* + t, V^u; l^u)}{\partial q}\} = 0$$

   Suppose that the good 2 is substitute to labor. Then, since $\frac{w^u * l^u}{w^s} \gg l^u$,$D^s(q, x^u; \frac{w^u * l^u}{w^s}) - D^u(q, x^u, l^u) \gg 0$. This implies that $t \gg 0$. Therefore, even under free trade , the consumption distortion is optimal.

2. Regressive case. $\lambda_1 = 0$ and $\lambda_2 \gg 0$.

   In this case, the result is opposite. It is optimal to provide the subsidy to the consumption of good 2.

3. First best case. $\lambda_1 = 0$ and $\lambda_2 = 0$.

   In this case, , we obtain $t = 0$ from equation(29).

4. Pooling case. $\lambda_1 \gg 0$ and $\lambda_2 \gg 0$

   In this case, it is difficult to decide the sign of t *a priori.*

Proposition 3 *When a utility function is not weakly separable between goods and labor and the Pareto-optimal progressive income tax system is 'redistributive,' then imposing a commodity tax on a good which is substitute to labor is optimal under free trade. If the progressive tax system is regressive, providing a subsidy to a good which is substitute to labor is optimal under free trade. If the progressive income tax system is first best, then the commodity tax or subsidy is zero.*

Next, we examine the effect of increasing the tariff from zero when the utility function is not weakly separable under a progressive income tax system. From the envelope theorem, the effect of increasing the tariff from zero is $\left.\frac{\partial \bar{L}}{\partial \sigma}\right|_{\sigma=0}$. Assuming that the income tax system is 'redistributive,' we obtain

$$\left.\frac{\partial \bar{L}}{\partial \sigma}\right|_{\sigma=0} = -\lambda_1 * \frac{\partial V^{su}(q, x^u; \frac{w^u * l^u}{w^s})}{\partial(\frac{w^u * l^u}{w^s})} * \Omega_\sigma * l^u \gg 0 \tag{30}$$

Equation (30) shows that the effect of the increasing the tariff with non-weakly separable utility function is same as the effect with weakly separable utility function.

Proposition 4 *Proposition 1 holds with non-weakly separable utility function*

Finally we characterize the optimal tariff and the optimal commodity tax when the utility function is not weakly separable. Suppose that the progressive income tax system is redistributive. Then, from the Lagrangian $\bar{L}$, optimal commodity tax is characterized as

$$\frac{\partial \bar{L}}{\partial t} = \lambda_1 * \frac{\partial V^{su}(q, x^u; \frac{w^u * l^u}{w^s})}{\partial x^u} * \{D^s(q, x^u; \frac{w^u * l^u}{w^s}) - D^u(q, x^u; l^u)\}$$
$$+\lambda_3 * (t + \sigma) * \{\frac{\partial h(p^* + t, V^s; l^s)}{\partial q} + \frac{\partial h(p^* + t, V^u; l^u)}{\partial q}\} = 0 \tag{31}$$

Optimal tariff is characterized from the following equation:

$$\frac{\partial \bar{L}}{\partial \sigma} = -\lambda_1 * \frac{\partial V^{su}(q, x^u; \frac{w^u * l^u}{w^s})}{\partial(\frac{w^u * l^u}{w^s})} * \Omega_\sigma * l^u - \sigma * \lambda_3 \frac{\partial y_2}{\partial \sigma} = 0. \qquad (32)$$

Therefore, both production distortion and consumption distortion are optimal.

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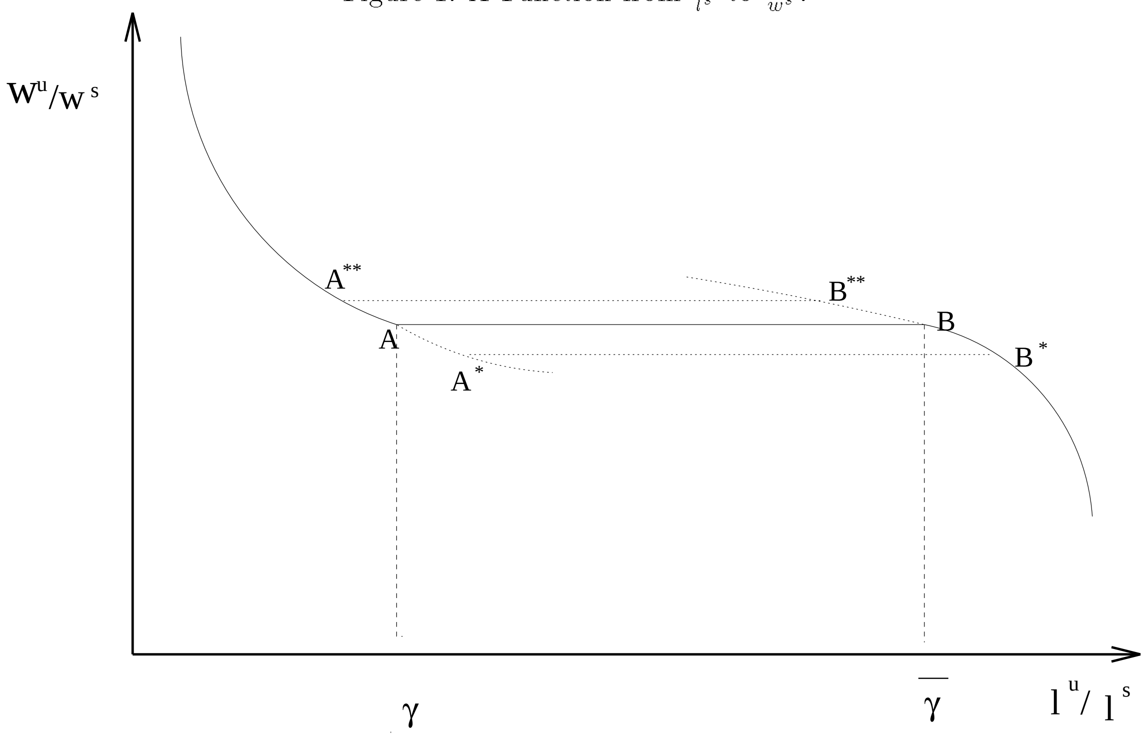
Figure 1: A Function from $\frac{l^u}{l^s}$ to $\frac{w^u}{w^s}$.



* If the government increases the tariff, then the line AB shifts to the line $A^{**}B^{**}$. If the government decreases the tariff, then the line AB shifts to the line A^*B^*.

Figure 2: Effect of A Tariff with or without commodity tax adjustment

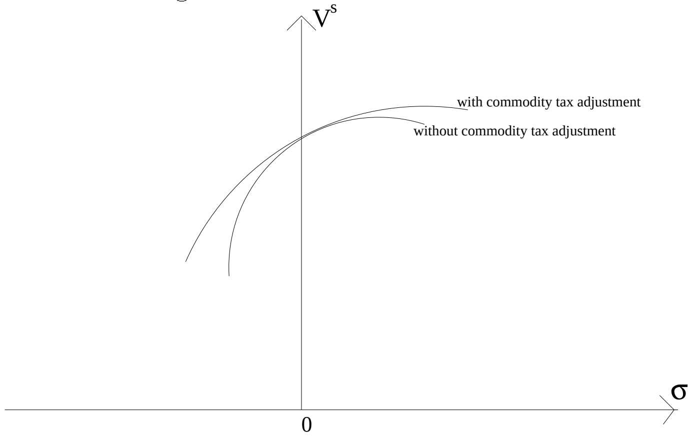


Figure 3: Effect of the tariff in the first best case

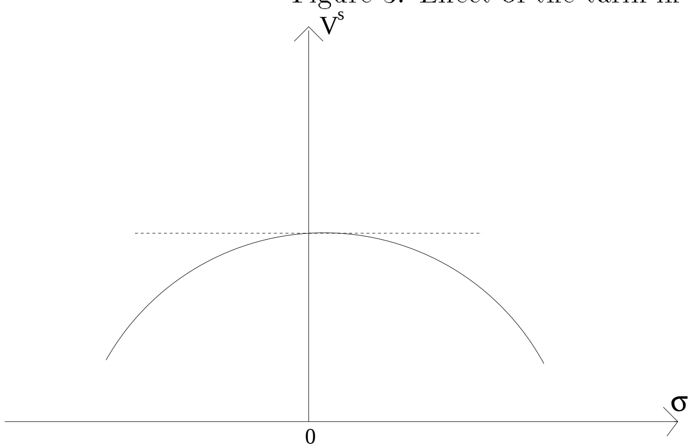
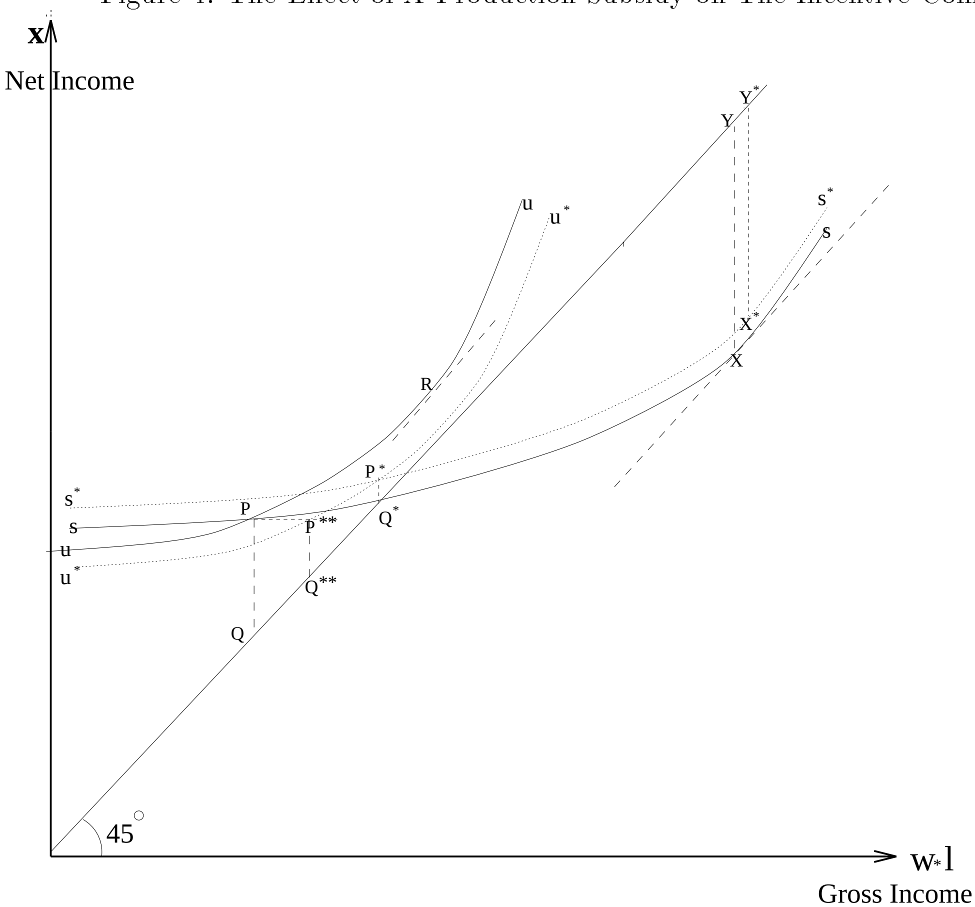


Figure 4: The Effect of A Production Subsidy on The Incentive Compatibility



[2] u and s are the indifference curves of the unskilled workers and skilled workers, respectively. At the point R, the slope of the indifference curve of the unskilled workers is 45° and the amount of the subsidies to the unskilled workers is minimized at R.